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Proceedings of Symposium on Ultrasonic Electronics, Vol. 30 (2009) pp. 91-92
18-20 November, 2009

Coherence Analysis of Difference Frequency Sounds Generated by Multiple Pairs of Primary Waves

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1. Introduction

The high power underwater parametric arrays have been used extensively in underwater acoustic communication, target detection, seabed classification and ocean monitoring[1)]. The conventional methods for the generation of the Difference-Frequency (DF) sound beam from a parametric array are generally based on the nonlinear interactions of one pair of fundamental waves with a proper frequency difference and high sound pressure intensity. However, to achieve the DF sound intensity high enough, a fair amount of radiated sound power corresponding to the single pair of primary waves is needed in practical applications. The ample power radiation of primary sound fields will result in the strongly nonlinear attenuation[2)], acoustic cavitation in seawater medium and high intensity interference. Meanwhile, the high-cost and complex power amplifier circuits are needed to obtain a large power output.

Aimed at these disadvantages from the large power radiation, we proposed a novel DF sound field generation mehtod of expoiting the coherent accumulation of multiple DF sounds generated by multiple pairs of primary waves with the equal frequency difference. The schematic diagram of this method is shown in **Fig.1**. This method lowers the radiation power in each frequency sub-band and maintains a high DF sound intensity . Based on the simplified algorithm of the DF sound field generated by one pair of primary waves[3,4)], we calculate the coherent accumulation of the DF sounds generated by multiple pairs of primary waves and analyze the sound pressure amplitude and phase bias among these DF sounds. Simulation results demonstrate the multiple DF sounds using this method can achieve a perfect coherent accumulation.

2. Coherence analysis of multiple DF sounds

The multiple pairs of fundamental waves can be radiated by a single plane source or a spherically focused array constructed by two intersecting sources. The total synthetic sound field of the multiple DF waves is denoted as

$$P_{DF,total} = \sum_{n=1}^{N} p_{l_n m_n}^{(n)}(\xi,\eta,\tau_n)$$

$$= \sum_{n=1}^{N} \mathrm{Re}\left\{-(l_n+m_n)^2 \times p_{n1}p_{n2} \times \left[\beta(\tilde{k}_n a)^2/\rho c^2\right] \times e^{-i(l_n+m_n)\tau_n} \times \overline{q_{l_n m_n}}(\xi,\eta)\right\} \times \cos(\phi_n - \phi_1) = \sum_{n=1}^{N} P_{DF,n} \times \cos(\phi_{DF,n} - \phi_{DF,1}) \quad (1)$$

where, $P_{DF,n}$ and $\phi_{DF,n}$ are the amplitude and phase of the DF sound generated by the n-th pair of fundamental sound waves. The variable $\overline{q_{l_n m_n}}(\xi,\eta)$ is given by

$$\overline{q_{l_n m_n}}(\xi,\eta) = \sum_{k=1}^{N_1}\sum_{j=1}^{N_2} A_k^{(n1)} A_j^{(n2)} \overline{q_{l_n m_n}}(\xi,\eta;k,j). \quad (2)$$

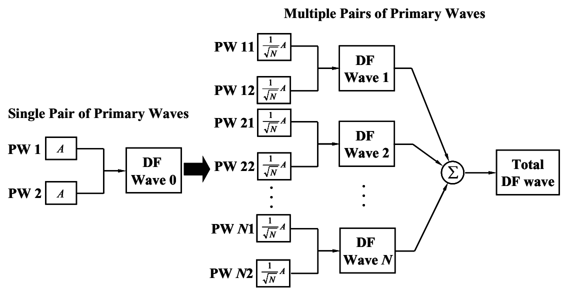

Fig.1. Schematic diagram of the new method

In these equations, $\xi = r/a$, $\eta = 2z/\tilde{k}_n a^2$ are the non-dimensional cylindrical coordinates, where r and z represent radial and axial coordinates respectively, and a is the radius of the radiator. $\tilde{k}_n = (\tilde{k}_{n1} + \tilde{k}_{n2})/2$, $l_n = \tilde{k}_{n1}/\tilde{k}_n$, $m_n = -\tilde{k}_{n2}/\tilde{k}_n$, $\tau_n = w_n t - \tilde{k}_n z$, $w_n = \tilde{k}_n c$, where $k_{ni} = 2\pi f_{ni}/c$, $(i = 1,2)$ is the wave number of the n-th pair of fundamental waves, $p_{ni} = \rho u_{ni} c$ is the characteristic pressure amplitude of the n-th pair of primary waves, and u_{ni} is the amplitude of vibration velocity, where ρ , c and β are the density, sound propagation velocity and nonlinear acoustic parameter of the medium. $A_k^{(n1)}$, $B_k^{(n1)}$, $A_j^{(n2)}$ and $B_j^{(n2)}$ are the Gaussian expansion coefficients of the n-th pair of primary waves. The variable $\overline{q_{l_n m_n}}(\xi,\eta;k,j)$ is written as

$$\overline{q_{l_n m_n}}(\xi,\eta;k,j) = \frac{1}{4r_1}\exp\left(-\frac{s_1}{r_1}\xi^2\right) \times \left[E_1\left(\frac{s_2}{r_1(r_1\eta + r_2)}\xi^2\right) - E_1\left(\frac{s_2}{r_1 r_2}\xi^2\right)\right], \quad (3)$$

where,

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$$r_1 = \left(l_n B^{(n2)}_{m_n j} + m_n B^{(n1)}_{l_n k}\right) + i\left(l_n + m_n\right)\eta B^{(n1)}_{l_n k} B^{(n2)}_{m_n j},$$

$$r_2 = \left(l_n B^{(n1)}_{l_n k} + m_n B^{(n2)}_{m_n j}\right)\eta - i\left(l_n + m_n\right),$$

$$s_1 = \left(l_n + m_n\right)^2 B^{(n1)}_{l_n k} B^{(n2)}_{m_n j}, \quad s_2 = -i\left(l_n + m_n\right) l_n m_n \left(B^{(n1)}_{l_n k} - B^{(n2)}_{m_n j}\right)^2,$$

$$B^{(n1)}_{l_n k} = \frac{B^{(n1)}_k}{l_n} + \frac{i}{\delta_n}, \quad B^{(n2)}_{m_n j} = \frac{B^{(n2)}_j}{m_n} + \frac{i}{\delta_n}, \quad \delta_n = \frac{2D}{a^2 \tilde{k}_n},$$

D is the geometric focal length of the spherically focused array and $E_1(z)$ is the exponential integral function. If we only care the second-order sound field of the DF component on the acoustic axis, the eq. (3) can be simplified as

$$\overline{q_{l_n m_n}}(0,\eta;k,j) = \frac{1}{4r_1}\ln\left(1 + \frac{r_1}{r_2}\eta\right). \qquad (4)$$

To analyze the coherence degree of multiple DF sounds, we need to calculate the relative phase bias among these DF sounds. The phase of the DF sound generated by the n-th pair of primary sounds is defined as

$$\phi_{DF,n} = -\left(l_n + m_n\right)\tau_n + \arctan\left[\frac{\mathrm{Im}\left(\overline{q_{l_n m_n}}(\xi,\eta)\right)}{\mathrm{Re}\left(\overline{q_{l_n m_n}}(\xi,\eta)\right)}\right]. \qquad (5)$$

3. Simulation Results

In the simulation experiments, we assumed the multiple pairs of primary sounds are radiated by a single large plane source with 1 m radius, so the focal length $D \to \infty$, and $1/\delta_n \to 0$. The physical parameters used in the simulations are listed as following: The characteristic pressure amplitudes of the n-th pair are defined as $p_{ni} = \left(1/\sqrt{N}\right) \times 10^{(P_{0i}/20) \times P_{ref}}$ Pa, $(i = 1,2)$, where, $P_{01} = P_{02}$ = 230 dB, P_{ref}= 1 μPa. The density ρ = 1000 Kg/m^3, sound speed c = 1500 m/s and nonlinear acoustic parameter β = 3.5. The Gaussian expansion coefficients are obtained from one paper of *Wen and Breazeale*[5]. The frequencies f_{n1} and f_{n2} of the n-th pair of primary waves are denotes as: $f_{n1} = f_o + (n-1) \times \Delta f$, $f_{n2} = f_o + (n-2) \times \Delta f$, where f_o and Δf is the initial frequency and equal frequency difference, respectively.

Assumed $t_0 = 0$, f_0 = 30 KHz, $\Delta f = 1$ KHz. N is increased from 1 to 20 and z from 20 m to 1000 m. The phase bias among multiple DF sounds is defined as $\Delta\phi_{DF,n} = \phi_{DF,n} - \phi_{DF,1}$. The phase bias curves of the axial DF sound pressure associated with the variables z and n are shown in **Fig.2**. As seen from Fig.2, it is obvious that the maximum value of the phase bias is less than 7°. These results demonstrate that multiple DF sound fields can achieve a perfect coherent accumulation.

Similar with the calculation of the phase bias, the amplitude bias among multiple DF sounds can be defined as $\Delta P_{DF,n} = P_{DF,n} - P_{DF,1}$. The amplitude bias curves of the axial DF sounds associated with the variables z and n is shown in **Fig.3**. It can be seen from Fig.3 that the maximum amplitude bias is smaller than 0.2 dB. These results demonstrate the synthetic sound field of multiple DF sounds can be accumulated effectivley.

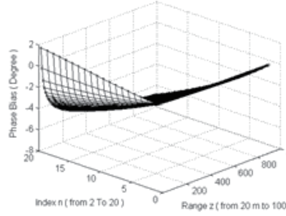

Fig.2. Phase bias curves associated with n and z

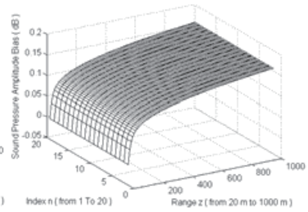

Fig.3. Amplitude bias curves associated with n and z

To compare the performance difference between a single pair and multiple pairs of primary waves, the amplitude difference of the DF sound field is defined as

$$\Delta P_{DF} = P_{DF,sp} - P_{DF,total}, \qquad (6)$$

where, $P_{DF,sp}$ is the DF sound amplitude generated by a single pair of primary waves. The amplitude difference curves associated with n are shown in **Fig.4**. As seen from Fig.4, the amplitude difference will increase with an increase of the pair number n.

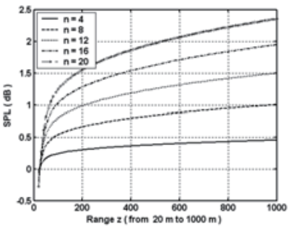

Fig.4. Amplitude difference curves associated with n

4. Conclusion

The presented DF sound field generation method of exploiting multiple pairs of primary waves can decrease the radiation power in each frequency sub-band and maintain the high sound intensity of the DF wave. Due to the small bias in the amplitude and phase among multiple DF sounds, it assures the perfect coherent accumulation of the multiple DF sounds.

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