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Acoustic Microrheology: Shear Moduli of Soft Materials Determined from Single Bubble Oscillations

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1. Introduction

Many materials such as gels and polymers are viscoelastic. One of the most important and frequently studied material properties of soft materials is the shear modulus. However, even in the simplest materials, properties are complex, and in contrast with most solids, the shear modulus can exhibit significant time, frequency and local position dependence. Thus, techniques that exploit the local viscoelastic properties of a material have great potential not only in material science but also in medical applications, where changes in the elasticity of tissue are often related to pathology.

A number of methods such as Dynamic Magnetic Resonance Elastography (MRE) [1,2], Acoustoelasticity [3], Supersonic shear imaging [4] or Vibro-acoustography [5] have been developed to evaluate the elastic properties of soft tissues. The general approach is to measure the response of material to an exitation force and use it to reconstruct the elastic parameters. In this article we introduce a new technique for determining the viscoelasticity of soft materials based on the oscillations of single bubbles injected into the material of interest.

2. Theory

It is well known that a bubble is a strong acoustic scatterer that exhibits a low-frequency resonance that depends on the elastic properties of the surrounding medium. Hence, by using a suitable model, measurements of the resonant frequency and damping rate can be used to accurately determine the complex shear modulus.

An air bubble with radius R_0 excited by a plane pressure wave $p\exp(-i\omega t+ikx)$, generates at distance r a spherical pressure wave $fp/r\cdot\exp(-i\omega t+ikr)$, where f is the scattering function given by

$$f(\omega,r)=\frac{R_0}{(\omega_0/\omega)^2-1+i\Gamma}. \qquad (1)$$

Here ω and k are angular frequency and wave vector of the pressure wave. ω_0 is known as the Minnaert resonance frequency and Γ is the damping rate, which includes viscous, thermal and acoustic radiation losses. For the case of a viscoelastic medium, good approximations for ω_0 and the viscous damping rate Γ_{vis} are given by

$$\omega_0^2=\frac{1}{R_0^2}\frac{3\gamma P_0+4\mu'}{\rho}, \qquad (2)$$

$$\Gamma_{vis}=\Gamma-\Gamma_{rad}-\Gamma_{therm}=\frac{4\mu''}{\rho R_0^2\omega^2}. \qquad (3)$$

Here γ is the ratio of specific heat capacities for air, ρ is the density of the material of interest, and $\mu=\mu'+i\mu''$ is its shear modulus. Formulae for the thermal and acoustic radiation losses can be found in [6]; details of the model are given in [7]. Using this model, the experimental values of the resonance frequency and damping rate of the oscillating bubble can be used to determine the material's complex shear modulus.

3. Materials and Methods

Experiments were performed on Agar gel of 2% concentration (Sigma). Bubbles from 0.4 to 1.2 mm in radius were injected into the materials with a syringe. The bubbles were then entrapped as the sol gelled. After gelling of the samples, small cubes were cut with a trimming blade. The bubble radii were measured by an optical imaging technique.

Measurements were performed in a water tank. First of all, the sample containing a single bubble was placed on top of a generating transducer with a central frequency of 100 kHz. The frequency of the continuous sinusoidal signal, produced by an arbitrary waveform generator, was slowly swept from 3 to 50 kHz with a step of 200 Hz. The signals at each frequency were averaged over 500 times in order to improve the quality of the data. The transmitted signals, p_{sam}, were detected by a hydrophone, amplified and recorded on digital oscilloscope. After that, the sample was carefully removed so that the hydrophone remained in the same position, and the procedure was repeated to obtain a reference signal, p_{ref}. Both the phase and the magnitude of acquired signals were calculated using fast Fourier transforms (FFT).

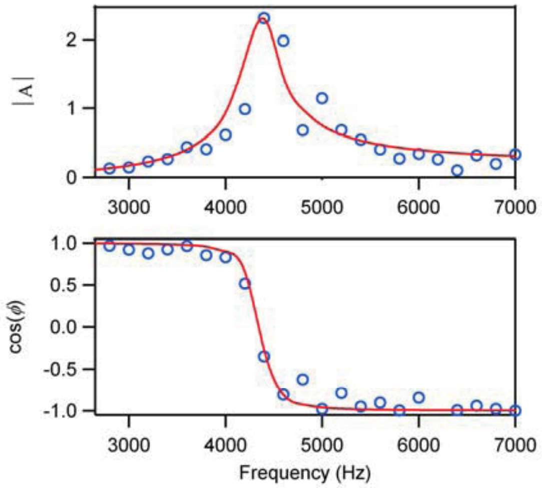

Fig. 1 Magnitude and cosine of the phase angle of $A(\omega)$ as a function of frequency.

4. Results and Discussion

As discussed by Leroy *et al* [8], the ratio of the extra pressure generated by the bubble oscillations ($p_{sam} - p_{ref}$) to the pressure p_{ref}, measured in the absence of the bubble

$$A(\omega) = \frac{p_{sam} - p_{ref}}{p_{ref}} \propto \frac{\omega^2}{\omega_0^2 - \omega^2 + i\omega\Gamma}, \quad (4)$$

can be used to determine the parameters ω_0, and Γ by applying a least squares method. **Figure 1** shows the magnitude and cosine of the phase angle of $A(\omega)$ as a function of frequency for a bubble with a radius equal to 0.92 mm in 2% Agar gel. Solid lines are fit with f_0 = 4350 Hz and Γ = 350 Hz, yielding μ' = 61.5 kPa and μ'' = 9.2 kPa.

If we apply the method described above to bubbles with different radii and hence different resonance frequencies, the frequency dependence of the shear modulus can be also obtained. **Figure 2** presents values of the shear modulus μ' measured from oscillations of individual bubbles as well as independent measurement at low frequencies (125,

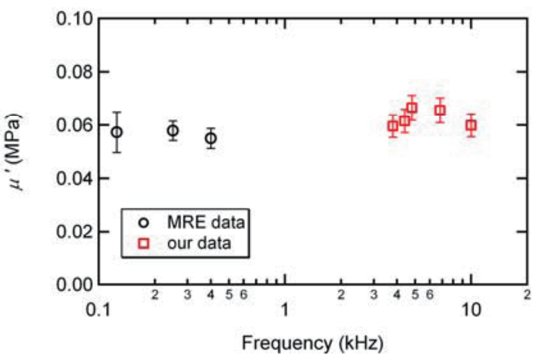

Fig. 2 Frequency dependence of the storage part of the shear modulus for 2% agar gel.

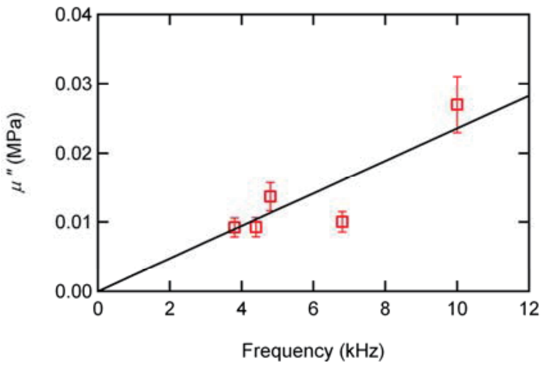

Fig. 3 Frequency dependence of the loss part of the shear modulus for 2% agar gel

250 and 400 Hz) obtained by MRE [2]. Similar values (not presented here) were obtained in the frequency range from 300 to 500 kHz using an acoustic reflection technique. It appears that there is no frequency dependence of μ' over a large frequency range. On the other hand, the loss part of the shear modulus μ'' is much smaller than μ' and increases with frequency (see **Fig. 3**). This acoustic technique is sufficiently sensitive to measure these small values of μ'', which are not detected by MRE.

5. Conclusions

In this paper, we demonstrate that the acoustic resonance of a single bubble can be used to accurately determine the local material properties of the medium in which the bubble is embedded. Because the resonance frequency is inversely proportional to the radius of the bubble, experiments on bubbles of different sizes enabled the frequency dependence of the complex shear moduli of the materials to be determined.

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