Proceedings of Symposium on Ultrasonic Electronics, Vol. 30 (2009) pp. 397-398
18-20 November, 2009

Analysis of Anisimkin's (Quasilongitudinal) Modes in Piezoelectric Plate
圧電板の Anisimkin（擬似縦波）モードの解析

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1. Introduction

Study of guided waves in a plate has a long history since Rayleigh and Lamb in 19th century. It has been a corner stone upon which such varied applications as seismology, Non-destructive testing and vibrating devices for frequency control and selection have been developed.

The behavior of dispersion curves of propagating waves is complex enough even in an isotorpic plate. The complexity increasaes in an anisotopic plate and there are still unexplored area to study. In fact, Ivan Anisimkin found numerically and verified experimentally peculiar modes propagating along the x (diagonal) axis in a quartz ST cut plate. [1)] They are quasilongitudinal (QL), that is dominat displacement is nearly uniform through the plate thickness with little horizontal and vertical shear displacements. Y. V. Gulyaev conducted further detailed studies. [2, 3)] Their analyses, however, were solely numerical.

This paper presents a mathematical anlysis and gives a formulus to yield "allowed" bands for QL-modes. The formulus also covers newly found quasishear (QS) modes, in which shear displacements are dominant with littel longitudinal displacement at the surfces of a plate.

Refined profiles of displcacements through the plate thickness are presented to solve a mistery of little jumps in their profiles of longitudinal displacement.

2. Analysis

For clarity, we present here pure elastic case of rotated Y cut of quartz, of which electromechanical coupling is small. A full treatment of piezoelectricity will be presented in a subsequent paper.

The stiffness matrix of rotated Y cut exhibits monoclinic symmetry shown in (1). Constants are normalized by c11, which is independent on rotation angle. The velocity of longitudinal wave is also independent on rotation angle.

$$cn=\begin{pmatrix} 1 & c12n & c13n & c14n & 0 & 0 \\ c12n & c22n & c23n & c24n & 0 & 0 \\ c13n & c23n & c33n & c34n & 0 & 0 \\ c14n & c24n & c34n & c44n & 0 & 0 \\ 0 & 0 & 0 & 0 & c55n & c56n \\ 0 & 0 & 0 & 0 & c56n & c66n \end{pmatrix} \quad (1)$$

Due to the symmetry, displacements can be written in the following forms, omitting a time factor.

For symmetric modes:

$$\begin{aligned} u &= U \cos\eta y \sin\xi x \\ v &= V \sin\eta y \cos\xi x \quad (2) \\ w &= W \sin\eta y \cos\xi x \end{aligned}$$

For anti-symmetric modes:

$$\begin{aligned} u &= U \sin\eta y \cos\xi x \\ v &= V \cos\eta y \sin\xi x \quad (3) \\ w &= W \cos\eta y \sin\xi x \end{aligned}$$

In order to satisfy the equation of motion, the following determinant shall be zero.

$$\begin{vmatrix} 1+c66n*x^2-Cpn^2 & (c12n+c66n)*x & (c14n+c56n)*x \\ (c12n+c66n)*x & c66n + c22n*x^2-Cpn^2 & c56n + c24n*x^2 \\ (c14n+c56n)*x & c56n+ c24n*x^2 & c44n*x^2+c55n -Cpn^2 \end{vmatrix} = 0 \quad (4)$$

where Cpn is phase velocity normalized by the velocity of longitudinal wave and:

$$x = \eta / \xi \quad (5)$$

For a given value of Cpn, (4) is a cubic equation of x^2, which yields three roots, xx1, xx2 and xx3 and its associated amplitude ratio of U, V and W.

Now the displacement consists of three similar terms corresponding to the above mentioned three roots with three coefficients. Traction free boundary conditions at the upper and the lower surfaces of the plate shall be satisfied and yield an equation in the following form, which yields dispersion curves.

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$$F(Cpn,\ \xi h) = 0 \quad (6)$$

where h is the half thickness of the plate and ξh is equal to $\pi H/\lambda$ in Anisimkin' notation.

3. Behavior in the vicinity of Cpn = 1

For QL-modes, behavior in the vicinity of Cpn = 1 is of particular interest.

For Cpn = 1, one root, xx1 becomes zero and the other two roots, xx2 and xx3, are given by the following reduced determinant.

$$\begin{pmatrix} c66n & (c12n+c66n) & (c14n+c56n) \\ (c12n+c66n) & c66n + c22n*xx-1 & c56n + c24n*xx \\ (c14n+c56n) & c56n+ c24n*xx & c44n*xx+c55n-1 \end{pmatrix} = 0 \quad (7)$$

Fig.1 shows these two roots as functions of rotated angle. It can be seen two roots are very close (but never touch) at the angle around 42.75° (132.75° in Anisimkin' notation).

Fig.2 shows ratios of displacements, u, v and w (U1, U3 and U2 in Anisimkin' notation) at the surface based on the solution of (4). Both ratios become zero at Cpn slightly higher than 1, which corresponds to a plateau of dispersion curve. Since a dominant component of the determinant in (4) is a single trigonometric function, such zero crossing points exist along ξh axis with regular interval. They are approximately given by:

$$\xi h \cong \frac{m\pi}{2\sqrt{xx}} \quad (8)$$

where m is an even integer for QL modes.

Fig. 3 shows a profile of displacements across the thickness for QL mode (m=1). They are smooth sinusoidal curves without any peculiar jump mentioned by Gulyaev [2].

4. Quasishear modes

When m is an odd integer, (8) yields still approximate solutions of (6), which may be called quasishear (QS) modes, because shear displacements are dominant with little longitudinal displacement at least at the surfaces of the plate as shown in Fig. 4. It is like a horizontal shear wave in an isotropic plate, but accompanies substantial longitudinal displacement inside.

References

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3. Y. V. Gulyaev, IEEE Trans. Ultrason. Ferroelectr. Freq. Contrl, **56**(2009)1042-1045.

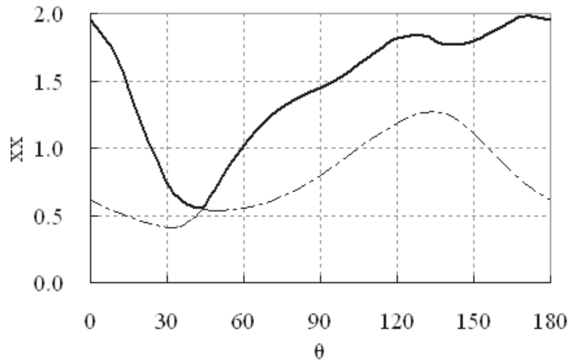

Fig. 1 xx2 and xx3, square of wave number across the thickness as functions of rotation angle.

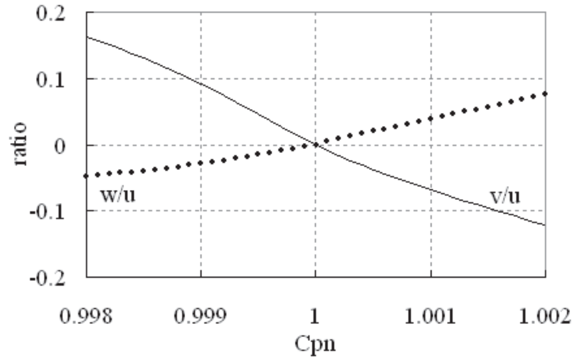

Fig.2 Ratios of shear displacements to longitudinal displacement at the surface. (QL,m=2)

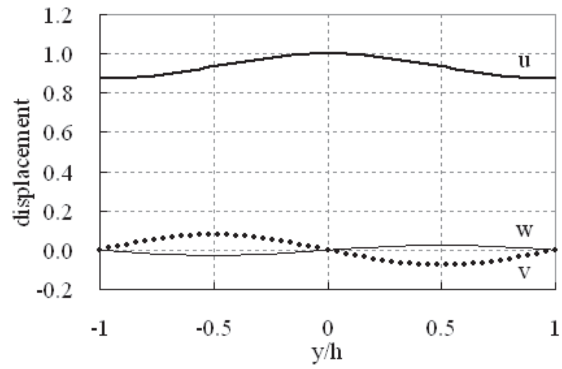

Fig. 3 Profile of displacements across the thickness for QL mode (m=2).

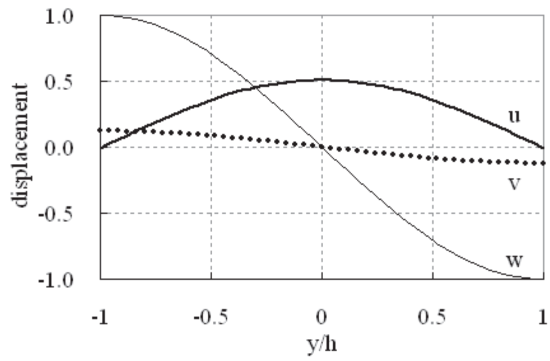

Fig. 4 Profile of displacements across the thickness for QS mode (m=1).