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Analysis of Nanomechanical Antenna Structure Resonator Using Transfer Matrix Method and Examination of its Frequency Gap

ナノメカニカル・アンテナ構造振動子の伝送行列法による解析とその周波数ギャップの検討

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1. Introduction

Nanomechanical antenna structure resonator with local vibrating forty elements on both side of central clamped-clamped boom has attracted considerable attention because the discrete displacement known as quantum one was observed at a frequency of 1.49 GHz at 110 mK. [1) 2)] We found the frequency gap in resonant frequency characteristics of this antenna structure resonator by calculating the resonant frequency of this structure using the Bernoulli-Euler equation.[3)] The resonant frequency characteristics, which shows a frequency gap in some resonant frequency band[3)4)], of this antenna structure resonator is similar to that of phononic crystals (PCs)[5)6)] with periodical masses coupled to one beam.

In this study, we analyzed the frequency gap of this antenna structure resonator using transfer matrix method with respect to a propagating flexural wave. Appling transfer matrix method to the repeated T-shaped unit of the antenna structure resonator, we calculated the attenuation and the dispersion relation between frequency and wave number of the propagating flexural wave using Bloch wave function under the periodic boundary condition of this unit and clarified the reason why the frequency gap occurred in the resonant frequency characteristics from the standpoint of the attenuation of this propagating flexural wave.

2. Analysis and discussion

Figure 1 shows that the antenna structure resonator consists of 21 repeated T-shaped units.[1)2)] We analysis the relationship between the integral constant $\mathbf{N}_{j-1}$ of T-shaped unit (j-1) and $\mathbf{N}_j$ of another neighboring unit (j) for the propagating flexural wave using transfer matrix method[5)6)] and it can be expressed as

$$\mathbf{N}_j = \mathbf{W}\mathbf{N}_{j-1}, \quad (1)$$

where W is 4×4 matrix obtained by the boundary conditions applied to two elements and one boom.[3)] In eq.(1), the integral constant matrix $\mathbf{N}_j$ is assumed to be

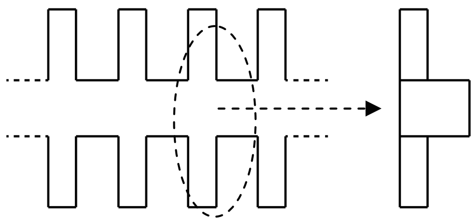

Fig. 1 Antenna structure resonator consists of 21 repeated T-shaped units.

$$\mathbf{N}_j = \lambda^j \mathbf{p}, \quad (2)$$

where λ is the diagonal matrix and $\mathbf{p}$ is a constant matrix.

Substituting eq. (2) into eq. (1), the eigenvalue equation of transfer matrix $\mathbf{W}$ is obtained to be

$$|\lambda - \mathbf{W}| = 0. \quad (3)$$

From eq.(3), we can then obtain the following eigenvalue equation as

$$\lambda^2 + a\lambda + b + a\frac{1}{\lambda} + \frac{1}{\lambda^2} = 0, \quad (4)$$

$$\begin{cases} a = -2(\cos\beta_j l + \cosh\beta_j l) + \frac{1}{2}\gamma_j(\sin\beta_j l - \sinh\beta_j l), \\ b = 2 + 4\cos\beta_j l \cdot \cosh\beta_j l - \gamma_j(\sin\beta_j l \cdot \cosh\beta_j l - \cos\beta_j l \cdot \sinh\beta_j l). \end{cases} \quad (5)$$

Equation (4) has explicitly four roots indicating by

$$\begin{cases} \lambda_1 = \dfrac{1}{\lambda_2} = \dfrac{1}{4}\left(D_+ + \sqrt{D_+^2 - 16}\right), \\ \lambda_3 = \dfrac{1}{\lambda_4} = \dfrac{1}{4}\left(D_- + \sqrt{D_-^2 - 16}\right), \end{cases} \quad (6)$$

$$D_\pm = -a \pm \sqrt{a^2 - 4b + 8}. \quad (7)$$

According to the Bloch theorem in periodic structure,[7)] the relationship between $\mathbf{N}_{j-1}$ and $\mathbf{N}_j$ is assumed to hold the following equation as

$$N_j = \exp(iq_k l) N_{j-1}, \quad (k = 1,2,3,4) \quad (8)$$

From eqs. (3) and (8), we can obtain the following eigenvalue equation expressed by

$$|\lambda - \exp(iq_k l)| = 0. \ (k = 1,2,3,4) \quad (9)$$

Using eq.(9), the four roots $\lambda_k (k = 1,2,3,4)$ can be expressed as

$$\lambda_k = \exp(iq_k l), \quad (10)$$

where $q_k l$ is the complex wave number.

We can calculate the attenuation and the dispersion relation between frequency and wave number $q_k l$ for the progressive flexural wave propagating through one of the repeated T-shaped unit of the antenna structure resonator under the periodic boundary condition from eqs. (5),(6),(7), and (10). Figures 2 and 3 show the attenuation and the dispersion relation between frequency and wave number $q_1 l$ and $q_2 l$, and $q_3 l$ and $q_4 l$, respectively. In Fig.2, $q_{2,i}l$ (denoted by thick dotted line in Fig. 2) is always positive, on the other hand, $q_{1,i}l$ (denoted by thin dotted line in Fig. 2) is also always negative, in the frequency band between 10 MHz and 3 GHz, so that $\exp(-q_{2,i}l)$ and $\exp(q_{1,i}l)$ functions become always attenuation ones. Therefore, the flexural waves indicated by λ_2 and λ_1 aren't excited in the frequency band between 10 MHz and 3GHz. In Fig. 3, $q_{3,i}l > 0$ and $q_{4,i}l < 0$ (denoted by thick dotted line and thin dotted line, respectively, in Fig. 3) are realized in the frequency band between 927 MHz and 1.197 GHz. Therefore, $\exp(-q_{3,i}l)$ and $\exp(q_{4,i}l)$ functions become attenuation ones in that frequency band. The flexural waves indicated by λ_3 and λ_4 show the existence of frequency gap that progressive and regressive waves aren't excited in the frequency band between 927 MHz and 1.197 GHz.
This result nearly coincides with the frequency gap between 763MHz and 1.197MHz of the antenna structure resonator calculated using eigenfunction of Bernoulli-Euler equation[3)]

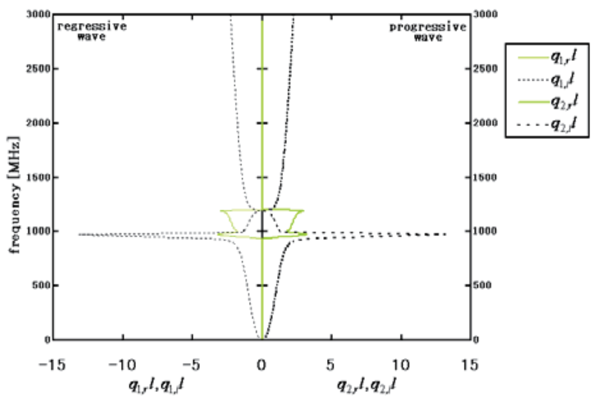

Fig. 2 The attenuation and dispersion relation between frequency and wave number $q_1 l, q_2 l$

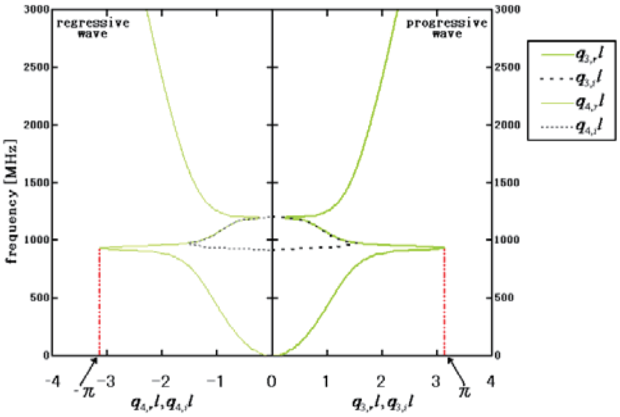

Fig. 3 The attenuation and dispersion relation between frequency and wave number $q_3 l, q_4 l$

3. Conclusion

We calculated the attenuation and dispersion relation of a progressive flexural wave propagating through the repeated T-shaped unit under periodic boundary condition using transfer matrix method. As our results of calculated frequency gap being almost similar to formerly obtained frequency gap [3)] of the antenna structure resonator using eigenfunction of Bernoulli-Euler equation, we can conclude that the antenna structure resonator with 40 elements on both side of boom is nearly equal to the repeated T-shaped unit under periodic boundary condition. In addition, the existence of attenuating flexural wave in the frequency band between 927 MHz and 1.197 GHz elucidates the appearance of frequency gap in resonant frequency characteristics of the antenna structure resonator because of its periodic structure having local vibration of elements.

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