

# Wideband Speaker Using Exponentially Tapered Piezoelectric Bimorph Actuators

指数関数に従って先が細い圧電バイモルフ振動子を用いた広帯域スピーカ

Huiuk Lee<sup>1†</sup>, Jungsoon Kim<sup>2</sup>, Moojoon Kim<sup>1</sup> and Kanglyeol Ha<sup>1</sup> (<sup>1</sup>Dept. Physics, Pukyong National Univ.; <sup>2</sup>Dept. Media Eng., Tongmyong Univ.)  
李熙旭<sup>1†</sup>, 金正順<sup>2</sup>, 金茂俊<sup>1</sup>, 河康烈<sup>1</sup> (<sup>1</sup>釜慶大 自然科学; <sup>2</sup>東明大 情通工)

## 1. Introduction

Recently, while the electronic devices are becoming slim style, the thin type speaker is demanded to design for the devices. The thin type moving-coil loud speaker has been developed for the mobile devices such as mobile phone, GPS, and DMB-TV. However, the speaker needs a massive magnet to induce the driving force for the low-range frequencies. To design the thin type speaker, piezoelectric speaker has been developed using a bimorph or a unimorph actuator<sup>1</sup>. Because of high quality factor of the piezoelectric vibrator, the frequency range of fidelity of the speaker is limited in narrow bandwidth. The characteristics of the piezoelectric bimorph actuator are changed according to its shape. In this study, the characteristics change by the shape is analyzed theoretically. The exponentially tapered piezoelectric bimorph actuators are fabricated, and the characteristics change are confirmed experimentally. The array of the differently tapered actuators could be driven in wideband frequency range.

## 2. Theory

As shown in Fig. 1(a), we assumed that the width of the piezoelectric bimorph, which an elastic material with thickness  $2t_m$  inserted into two piezoelectric plates, is changed exponentially. Poling direction of two piezoelectric plates is parallel with  $z$ -axis. If length  $l$  is long enough to comparing thickness  $t_p$  and width  $b$ , as shown in Fig. 2(b), the constitutive equation is written as follows<sup>2</sup>:

$$S_2 = s_{22}^E T_2 + d_{32} E_3 \quad (1)$$

$$D_3 = d_{32} T_2 + \varepsilon_{33}^T E_3 \quad (2)$$

where  $S_2$  is strain,  $s_{22}^E$  is elastic constant,  $T_2$  is stress,  $d_{32}$  is dielectric constant,  $E_3$  is electric field,  $D_3$  is electric density, and  $\varepsilon_{33}^T$  is permittivity under constant stress.

Including the tapered shape into the analysis of the bending mode<sup>3</sup>, the wave of equation can be obtained as following equation.

$$\frac{\partial^4 \xi_p}{\partial y^4} + 2\alpha \frac{\partial^3 \xi_p}{\partial y^3} + \alpha^2 \frac{\partial^2 \xi_p}{\partial y^2} - k_p^2 \xi_p - Q_V \alpha^2 = 0, \quad (3)$$

where,

$$k_p = \sqrt{\frac{2\omega^2(t_p \rho_p + t_m \rho_m)}{K_M}}, \quad Q_V = \frac{NV}{K_M}, \quad \alpha = \frac{1}{l} \ln\left(\frac{a}{b}\right),$$

$$N = (t_p + 2t_m) \frac{d_{32}}{s_{22}^E},$$

$$K_M = 2 \left[ Y_M \frac{t_m^3}{3} + \frac{1}{s_{22}^E} \frac{1}{1-k_{32}^2} \left[ \frac{t_p(t_p^2 + 3t_p t_m + 3t_m^2)}{3} - \frac{t_p(t_p + 2t_m)^2}{4} k_{32}^2 \right] \right],$$

To obtain the solution of displacement, following mechanical boundary conditions are applied to the differential equation.

$$\xi_p = 0 \ \& \ \frac{\partial \xi_p}{\partial y} = 0, \quad \text{at } y = 0,$$

$$M_p = 0 \ \& \ F_p = 0, \quad \text{at } y = l$$

From some manipulation, the displacement of the actuator is obtained as

$$\xi_p = \left\{ \frac{M_{c1}}{\Lambda_c} e^{-h_1 \frac{y}{2}} + \frac{M_{c2}}{\Lambda_c} e^{-h_2 \frac{y}{2}} + \frac{M_{c3}}{\Lambda_c} e^{-h_3 \frac{y}{2}} + \frac{M_{c4}}{\Lambda_c} e^{-h_4 \frac{y}{2}} - \frac{\alpha^2 N}{k_p^4 K_M} \right\} V$$

where,

$$h_1 = \alpha + \sqrt{\alpha^2 - 4k_p^2}, \quad h_2 = \alpha - \sqrt{\alpha^2 - 4k_p^2},$$

$$h_3 = \alpha + \sqrt{\alpha^2 + 4k_p^2}, \quad h_4 = \alpha - \sqrt{\alpha^2 + 4k_p^2}$$

$$\Lambda_c = \begin{bmatrix} e^{\frac{l}{2}(h_1+h_2)} (h_3-h_4)(h_1-h_2)h_3^2h_4^2 \\ + e^{\frac{l}{2}(h_1+h_3)} (h_3-h_1)^2h_2^2h_4^2 \\ - e^{\frac{l}{2}(h_1+h_4)} (h_4-h_1)^2h_2^2h_3^2 \\ - e^{\frac{l}{2}(h_2+h_3)} (h_3-h_2)^2h_4^2h_1^2 \\ + e^{\frac{l}{2}(h_2+h_4)} (h_4-h_2)^2h_1^2h_3^2 \\ + e^{\frac{l}{2}(h_3+h_4)} (h_3-h_4)(h_1-h_2)h_1^2h_2^2 \end{bmatrix} k_p^4,$$

$$A = e^{\frac{l}{2}h_2} (h_4-1)h_2h_3^2h_4^2 + e^{\frac{l}{2}h_3} (h_3-h_1)h_3h_2^2h_4^2 + e^{\frac{l}{2}h_4} (h_1-h_4)h_4h_2^2h_3^2,$$

$$B = e^{\frac{l}{2}(h_2+h_3)} (h_3-2\alpha)(h_2-h_3)h_4^2 - e^{\frac{l}{2}(h_2+h_4)} (h_4-2\alpha)(h_2-h_4)h_3^2 \\ + e^{\frac{l}{2}(h_3+h_4)} (h_1-2\alpha)(h_3-h_4)h_2^2,$$

$$C = e^{\frac{l}{2}h_1} (h_4-1)h_1h_3^2h_4^2 + e^{\frac{l}{2}h_2} (h_3-h_2)h_3h_1^2h_4^2 + e^{\frac{l}{2}h_3} (h_2-h_4)h_4h_1^2h_3^2,$$

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kimmj@pknu.ac.kr; kimj@tu.ac.kr

$$\begin{aligned}
D &= e^{\frac{l}{2}(h_1+h_3)}(h_3-2\alpha)(h_1-h_3)h_4^2 - e^{\frac{l}{2}(h_1+h_4)}(h_4-2\alpha)(h_1-h_4)h_3^2 \\
&\quad + e^{\frac{l}{2}(h_2+h_4)}(h_2-2\alpha)(h_3-h_4)h_1^2, \\
E &= e^{\frac{l}{2}h_1}(h_3-h_1)h_1h_2^2h_4^2 + e^{\frac{l}{2}h_2}(h_2-h_3)h_2h_1^2h_4^2 + e^{\frac{l}{2}h_4}(h_1-h_2)h_1^2h_2^2h_4, \\
F &= e^{\frac{l}{2}(h_1+h_2)}(h_3-2\alpha)(h_2-h_1)h_4^2 - e^{\frac{l}{2}(h_1+h_4)}(h_1-2\alpha)(h_4-h_1)h_2^2 \\
&\quad + e^{\frac{l}{2}(h_2+h_4)}(h_2-2\alpha)(h_4-h_2)h_1^2, \\
G &= e^{\frac{l}{2}h_1}(h_4-h_1)h_1h_2^2h_3^2 + e^{\frac{l}{2}h_2}(h_2-h_4)h_2h_3^2h_1^2 + e^{\frac{l}{2}h_3}(h_1-h_2)h_1^2h_2^2h_3, \\
H &= e^{\frac{l}{2}(h_1+h_2)}(h_4-2\alpha)(h_2-h_1)h_3^2 - e^{\frac{l}{2}(h_1+h_4)}(h_1-2\alpha)(h_3-h_1)h_2^2 \\
&\quad + e^{\frac{l}{2}(h_2+h_3)}(h_2-2\alpha)(h_3-h_2)h_1^2, \\
M_{c1} &= e^{\frac{l}{2}h_1} \left\{ \alpha^2 A + 4Bk_p^4 \right\} \frac{N}{K_M}, \quad M_{c2} = -e^{\frac{l}{2}h_2} \left\{ \alpha^2 C + 4Dk_p^4 \right\} \frac{N}{K_M}, \\
M_{c3} &= -e^{\frac{l}{2}h_3} \left\{ \alpha^2 E + 4Fk_p^4 \right\} \frac{N}{K_M}, \quad M_{c4} = e^{\frac{l}{2}h_4} \left\{ \alpha^2 G + 4Hk_p^4 \right\} \frac{N}{K_M}.
\end{aligned}$$

From the relation between electric current and voltage input admittance of the bimorph is given by

$$\begin{aligned}
Y &= j\omega b \frac{d_{32}}{s_{22}^E \Lambda_c} \left( \frac{t_p + 2t_m}{2} \right) \\
&\quad \times \left\{ \frac{h_1^2}{h_2} M_{c1} \left( e^{\frac{h_2 y}{2}} - 1 \right) + \frac{h_2^2}{h_1} M_{c2} \left( e^{\frac{h_1 y}{2}} - 1 \right) \right\} \\
&\quad \times \left\{ \frac{h_3^2}{h_4} M_{c3} \left( e^{\frac{h_4 y}{2}} - 1 \right) + \frac{h_4^2}{h_3} M_{c4} \left( e^{\frac{h_3 y}{2}} - 1 \right) \right\} \\
&\quad + j\omega b \frac{\epsilon_{33}^{LS}}{t_p} \frac{2}{\alpha} (e^{\alpha l} - 1)
\end{aligned} \quad (4)$$

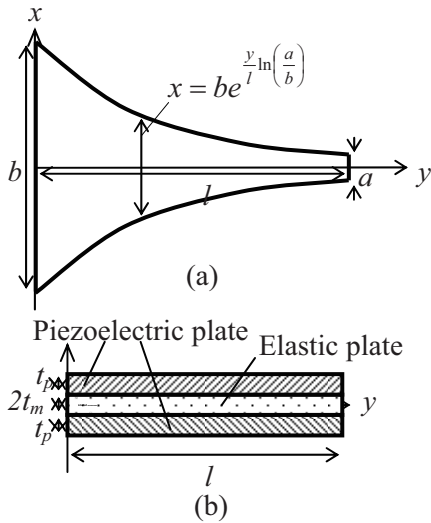
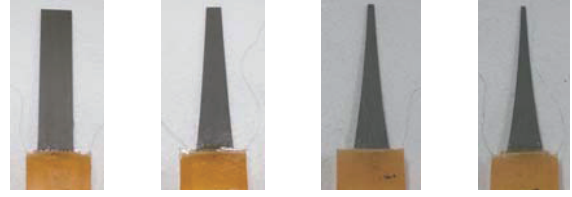


Fig. 1 Calculation model for bimorph actuator

### 3. Experiment and Results

To confirm the characteristic change by the shape, four piezoelectric bimorph actuators are designed as shown in **Fig. 2**, and data about the actuators are listed in **Table 1**. In calculation, the elastic material between the piezoelectric plates is ignored, and the mechanical loss is estimated as  $\tan \delta = 0.012$ . The four bimorph actuators are fabricated using CNC milling machine with CAD/CAM program, and one ends of them are

mounted by epoxy to satisfy the fixed-end condition. The results of input admittance are shown in **Fig. 3**. In this figure, lines show the theoretical results and scattered marks are measured one.



(a)  $\alpha=0$  (b)  $\alpha=11.1$  (c)  $\alpha=17.6$  (d)  $\alpha=25.8$

Fig. 2 Photograph of tapered piezoelectric bimorphs

Table 1 Design factors of bimorph actuators

	$\rho$ (kg/m <sup>3</sup> )	$s_{22}^E$	$k_{32}$	$b$ (mm)
	$7.370 \times 10^3$	$1.6690 \times 10^{-11}$	0.3861	12.7
	$\epsilon_{33}^T$	$d_{32}$	$l$ (mm)	$t_p$ (mm)
	$3.2669 \times 10^{-8}$	$2.8123 \times 10^{-10}$	62.45	0.44
	$T_{x1}$	$T_{x2}$	$T_{x3}$	$T_{x4}$
$l$ (mm)	12.7	6.3	4.2	3.2

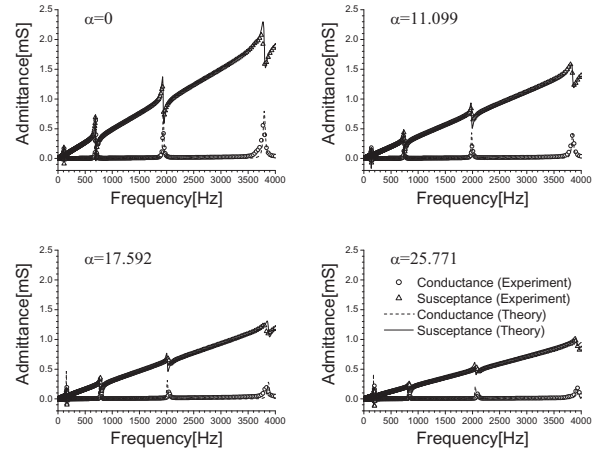


Fig. 3 Input admittance change with shape

### 4. Conclusion

A theoretical analysis method is derived for the piezoelectric bimorph actuator of exponentially tapered shape. The differently tapered four bimorph actuators are fabricated and their characteristics are investigated. The experimental results coincide well with theoretical results. Using the analyzing method, the piezoelectric bimorph actuator with proper characteristics could be designed. We can expect that a wideband acoustic speaker could be designed with the array of the actuators with different resonant.

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