3P5-30

Proceedings of Symposium on Ultrasonic Electronics, Vol. 30 (2009) pp. 581-582
18-20 November, 2009

Determination of bovine bone anisotropic stiffness using ultrasonic data in two orthogonal planes

Quentin Grimal[1,2], Kazufumi Yamamoto[3], Tomohiro Nakatsuji[3], Mami Matsukawa[3] and Pascal Laugier[1,2] ([1] UPMC Univ Paris 06, UMR 7623, LIP, F-75005, Paris, France; [2] CNRS, UMR 7623, Laboratoire d'Imagerie Paramétrique, F-75005, Paris, France, [3]Laboratory of Ultrasonic Electronics, Doshisha University, Kyoto, Japan)

1. Introduction

Quantitative ultrasound clinical devices measure the speed of sound (SOS) in the MHz-range. SOS has been shown to reveal useful information on bone quality. SOS is related to the microstructure (porosity) and to the amount and orientation of mineral cristals. Bovine bone which has a low volume fraction of pores compared to human bone is a relevant model to investigated the relationship between mineral cristals and SOS[1]. In addition, in bovine bone both Haversian and plexiform tissue patterns exist, the occurrence of which depend on age of specimen and anatomical location.

Several authors have attempted to characterize the elastic anisotropy of cortical bone. There are some experimental and theoretical evidence that Haversian pattern corresponds to a transversely isotropic material at the macroscale[2, 3] while the plexiform pattern corresponcs to an orthotropic material at the macroscale[4]. However it is unknown how close to ideal symetry is the actual bone symmetry. The possible deviations from ideal symmetry at the macroscopic scale have not been tested due to a experimental limitations: available SOS measurements were obtained exclusively from bone cubes cut according to the speculated symmetry axes. In other words the classical method for the measurement of elastic anisotropy in bone does not provide enough information to test the symmetry hypothesis.

The present work takes advantage of a recently obtained set of data[1] giving SOS as a function of angle. The objective of the present work is twofold: 1) to test how close to hexagonal or orthotropic is the symetry of bovine bone; 2) to determine several terms of the elasticity tensor with a method robust to measurement uncertainties.

2. Method

Four cylindrical samples of diameter 11 mm with axis the radial direction of bone were obtained from the mid-diaphysis of three bovine femurs in the anterior (A), posterior (P), medial (M) and lateral (L) parts. (In this abstract only the results concerning one sample of the anterior part are presented.) Through transmission SOS was measured perpendicular to the cylinder axis with a conventional pulse system centered at 1 MHz[5]. SOS measurements were obtained for a collection of rotation angles (rotation step: 3 degree) of the cylinder axis. In a second step, one other cylindrical sample with axis the tangential direction of the bone organ was cut into each of the previously obtained cylinders. SOS was measured in this second set of cylinders using the same protocol as described above. Doing so, two sets of SOS measurements versus angle are obtained for each anatomical position (A,P,L,M) in the axial-tangential (AT) and axial-radial (AR) planes.

In order to account for small deviations of the actual material symmetry axes from the axes of the working frame based on bone organ anatomy, the origin of the angle coordinate was chosen on the apparent symmetry axis of experimental SOS vs. angle polar plot.

The velocity of quasi-longitudinal waves in an orthotropic material for each angle relative to its natural basis can be derived from Christoffell equations. In the two planes corresponding to the SOS measurements, SOS versus angle is given by explicit analytical formulas[6]. These indicate that propagation of longitudinal waves in the two measurement planes involves 7 of the 9 stiffness coefficients (all but C_{12} and C_{66}) that define the elastic behavior of an orthotropic material.

A sensitivity analysis was performed to test dependence of the theoretical velocities with respect to variations of stiffness coefficients around initial values which correspond to typical bone elasticity values from the literature: $C_{11} = 24$; $C_{22}= 23$; $C_{33}= 34$; $C_{13}= 10$; $C_{23}= 10$; $C_{44}= 8$; $C_{55}= 8$ (values in Gpa). Mass density was taken to be 2000 kgm^{-3}.

The value of the 7 involved stiffness coefficients that minimize the distance between the experimental data and theoretical SOS derived from the ideal orhotropic material in its natural basis were determined by solving an inverse problem. Different objective functions were considered making use of assumptions on values of stiffness coefficients. The objective functions were minimized using the simplex search method implemented in the fminsearch MATLAB function.

3. Results

Based the sensitivity analysis, it is expected that C_{33} be well determined from on experimental SOS at angles 0 and π; C_{11} and C_{22} be well determined from on SOS at $\pi/2$ and $3\pi/2$. Unfortunately the effect on the velocity of C_{13} and C_{55} on the one hand and C_{23} and C_{44} on the other hand are very similar; accordingly it will be difficult to determine them independently.

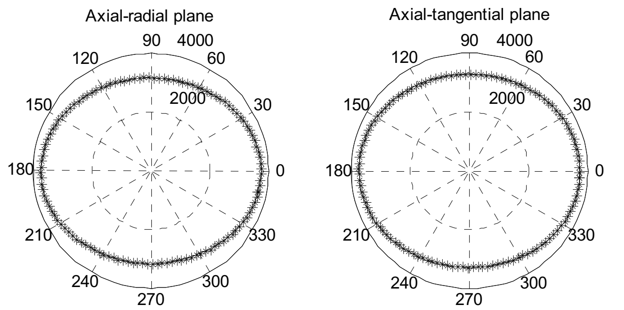

Figure 1 polar plot of experimental (*) vs modeled (continuous line) velocities for a sample of the anterior part of a bovine femur. Angle coordinate θ=0 corresponds to the bone organ axis (axial direction).

The elementary sum of square cost function has at least one minimum for unrealistic values of elasticity. Nevertheless for the latter the fit of the experimental data with model velocities assessed visually was very good. More elaborated cost functions with regularization terms were tested . Regularization was based on the following assumptions: (1) coefficients should be positive; (2) non diagonal coefficients should be close in values (as observed in available data). In addition, experimental data at specific angles were used preferentially for the determination of specific coefficients, based on the sensitivity study.

In this preliminary work, the inverse problem was solved for one bone sample of the anterior part. The following elasticity constants were determined (GPa): C_{11}=20.1, C_{13}=11.3, C_{22}=21.6, C_{23}=10.4, C_{33}=27.8, C_{44}= 6.9, C_{55}= 5.4. The obtained fit of experimental data by the model is shown in Fig. 1.

4. Discussion

Preliminary results were obtained for one set of experimental data giving SOS vs. angle in two orthogonal planes. The 7 determined values of elasticity are reasonable from a physical point of view and are in the range of published data for bovine bone. The results are consistent with an orthotropic model for bovine bone: the evolution of SOS with angle in the AT- and AR-planes can be fully explained with orthotropic symmetry, once the sample data is expressed in an hypothesized natural basis. With the available data set, coefficients C_{33}, C_{11} and C_{22} can be determined accurately and their determination is robust to experimental uncertainties and small errors in the hypothesized natural basis. Furthermore their determination is not sensitive to the choice of the cost function. Other coefficients are coupled and the determined values may not be unique for a given data set. Nevertheless, with a regularization that only includes physically relevant assumptions, reasonable physical values for all coefficients could be determined. An alternative solution to regularize the inverse problem would be to couple the stiffness tensor determination to a micromechanical model of bovine bone elasticity.

1. Yamamoto, K., Y. Yaoi, J. Grondin, Q. Grimal, Y. Yamato, T. Yanagitani, M. Matsukawa, P. Laugier, Y. Yamazaki, and A. Nagano. *Ultrasonic wave properties in the bone axis direction of human and bovine cortical bone*. in *International Congress on Ultrasonics*. 2009. Santiago, Chili.
2. Yoon, H.S. and J.L. Katz, *Ultrasonic wave propagation in human cortical bone-I. Theoretical considerations for hexagonal symmetry.* Journal of Biomechanics, 1976. **9**: p. 407-412.
3. Yoon, H.S. and J.L. Katz, *Ultrasonic wave propagation in human cortical bone--II. Measurements of elastic properties and microhardness.* Journal of Biomechanics, 1976. **9**(7): p. 459-64.
4. Katz, J.L., H.S. Yoon, S. Lipson, R. Maharidge, A. Meunier, and P. Christel, *The effects of remodeling on the elastic properties of bone.* Calcif Tissue Int, 1984. **36**(Suppl 1): p. S31-6.
5. Mizuno, K., M. Matsukawa, T. Otani, M. Takada, I. Mano, and T. Tsujimoto, *Effects of structural anisotropy of cancellous bone on speed of ultrasonic fast waves in the bovine femur.* IEEE Trans Ultrason Ferroelectr Freq Control, 2008. **55**(7): p. 1480-7.
6. Auld, B.A., *Acoustic Fields and Waves in Solids*. 1973, New York: John Wiley and Sons.