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Examination on Accurate and Fast Algorithms Using Compact FD-TD Acoustic Wave Simulation

コンパクト差分を用いた時間領域音場解析における高精度化と高速化の検討

Takeshi Yoda[1†], Kan Okubo[1], Norio Tagawa[1] and Takao Tsuchiya[2] ([1]Facult. System Design, Tokyo Met. Univ.; [2]Facult. Sci. Eng., Doshisha Univ.)
要田剛志 [1‡], 大久保寛 [1], 田川憲男 [1], 土屋隆生 [2]（[1]首都大　システムデザイン; [2]同志社大　理工）

1. Introduction

To date, time domain numerical analysis of acoustic fields has become investigated widely as a result of recent computational progress[1-6]. Some techniques have been proposed as an acoustic field calculation method; the finite difference time domain (FD-TD) method[1-3] is very widely used for time domain numerical analysis. Many results using numerical analyses of sound propagation have been reported by FD-TD method.

The standard FD-TD method based on Yee's algorithm, however, causes numerical dispersion error due to using second order finite difference (FD) approximation. To overcome this problem, FD-TD methods using higher order spatial FD have been proposed. Moreover, compact FD was developed as a more accurate schemes for numerically solving differential equations[7].

In the FD-TD simulation, it determines the calculation accuracy how to calculate the value of the spacial derivatives $f'(i\Delta x)$ on the discretized grids. Higher order FD-TD methods yield superior accuracy in exchange for a little more complicated formulation. Especially, a tridiagonal or pentadiagonal linear system must be solved to calculate the derivatives of the compact FDs. The implementation of pentadiagonal is quite complicated. Therefore, this calculation cost is a bottleneck of the development of acoustic simulation using compact FD-TD method.

In this study, we propose the fast and efficient calculation method of compact FDs employing a recursive filter. The recursive filter is based on z-transformation and its recursive filtering algorithm[8].

2. Acoustic Simulation Using Compact FD-TD Method

The compact FDs are obtained by relation equations of the surround values and their derivatives, which are given by the following equation:

$$\beta f'_{i-2} + \alpha f'_{i-1} + f'_i + \alpha f'_{i+1} + \beta f'_{i+2} = c\frac{f_{i+5/2} - f_{i-5/2}}{5\Delta} + b\frac{f_{i+3/2} - f_{i-3/2}}{3\Delta} + a\frac{f_{i+1/2} - f_{i-1/2}}{\Delta}. \quad (1)$$

Here, Δ is the grid size of FD-TD calculations.

As compared with the central FDs of the same order, which use only surround values, the compact FDs provide higher accuracy [7]. That is the parameters of Eq. (1) are determined by relation equations of values and their derivatives using Taylor series.

For example, relation equations for the fourth-order and eighth-order compact FDs are given by Eqs. (2) and (3), respectively

$$f'_{i-1} + 22f'_i + f'_{i+1} = \frac{12}{11}\frac{22(f_{i+1/2} - f_{i-1/2})}{\Delta}, \quad (2)$$

$$f'_{i-2} + \frac{\alpha}{\beta}f'_{i-1} + f'_i + \frac{\alpha}{\beta}f'_{i+1} + f'_{i+2} = b\frac{f_{i+3/2} - f_{i-3/2}}{3\beta\Delta} + a\frac{f_{i+1/2} - f_{i-1/2}}{\beta\Delta}, \quad (3)$$

where α=6114/25669, β=(354α-75)/2614, a=(37950-39275α)/31368, b=(65115α-3550)/20912. As shown in these equations, the fourth-order and eighth-order compact FDs are respectively required to compute tridiagonal and pentadiagonal matrices.

3. Fast Calculation Method of Compact FD-TD

We present the efficient calculation method of compact FDs employing a recursive filter. Now, by assuming

$$(f_i)_{fd} = c\frac{f_{i+5/2} - f_{i-5/2}}{5\Delta} + b\frac{f_{i+3/2} - f_{i-3/2}}{3\Delta} + a\frac{f_{i+1/2} - f_{i-1/2}}{\Delta}, \quad (4)$$

Eq. (1) is rewritten for deconvolution form as

$$f_i' = (f_i)_{fd} * h_{fd}^{-1}. \qquad (5)$$

Then, h_{fd}^{-1} is a symmetrical stable filter. Therefore, the efficient implementation using recursive filtering algorithm can be employed to calculate derivatives in the compact FDs according to:

$$(f_i')^+ = (f_i)_{fd} + z_k (f_{i-1}')^+, \qquad (i = 2,\ldots,N)$$

$$(f_i')^- = z_k \left((f_{i-1}')^- - (f_{i-1}')^+ \right), \qquad (i = N-1,\ldots,1)$$

This makes it possible to solve pentadiagonal (tridiagonal) scheme by using the recursive filter two times (one time). In interpolation of order 2*K*+1, by using *z* transform for Eq. (4), the inverse basis function h_{fd}^{-1} can be a symmetrical stable filter of order 2*K* with specific pole values z_k (k = 1, …, K). Its similar implementation for B-spline interpolation using recursive filtering algorithm has be also provided.

Now, the specific pole values z_k of the forth order and sixth order compact FDs, which is used for the recursive filtering algorithm, are provided as follows:

i) the forth order compact FD:

z_1 = -0.0455488499

ii) the eighth order compact FD:

z_1= -0.2363652294, z_2 = -0.0160419595.

Thus, by employing the recursive filtering algorithm for the compact FDs, we can obtain the efficient calculation method of highly accurate derivatives which has almost as large computational cost as the conventional methods.

4. Results and discussion

We show the numerical results obtained using the above compact FD-TD analysis by recursive filtering algorithm. Calculation parameters are: the direction of acoustic field propagation, $\pm x$ (1D analysis); grid size, Δx = 0.05 m; number of grid points, Nx =20000.

Figure 1 shows the sound pressure distribution obtained using FDTD analysis at t = 0.81 s, where ρ =1.21 kg/m^3 and K = 1.4236×10^5 Pa. Here, the initial pressure at t = 0 is given as $p = e^{-\alpha(x-x0)^2}$ [N/m^2]. (48≤ x ≤500) In this equation, $\alpha = 1/20$ and $x_0 = 500$. The results obtained using the eighth order compact FD-TD method agree well with the analytical solution. Here, in this calculation, we use time step Δt=1.2×10^{-5} s (i.e., CFL=0.0823). On the other hand, other methods cause numerical dispersion error.

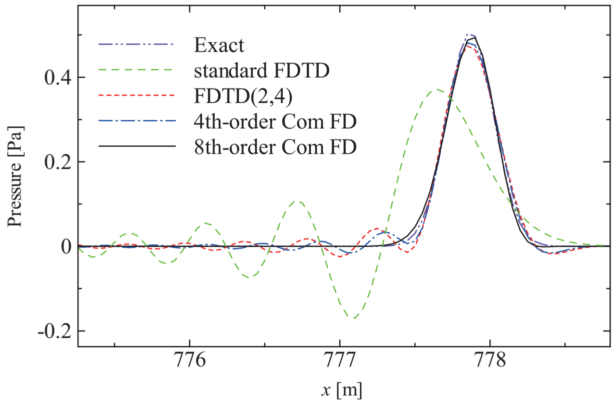

Fig. 1 Distribution of the sound pressure at t = 0.81 s; illustrates standard FDTD (Δt =1.35×10^{-4} , CFL=0.926), FDTD(2,4) (Δt=2.7×10^{-5} ,CFL=0.185), 4th order compact FD-TD (Δt=2.4×10^{-5}, CFL=0.15), 8th order compact FD-TD (Δt=1.2×10^{-5}, CFL=0.0823.

Recursive filtering algorithm of fourth order compact FD approximately has a complexity of 6 Nx flops (3Nx multiplications + 3Nx adds). This algorithm doesn't require matrix decomposition processes like Gaussian elimination or LU factorization.

5. Conclusion

This study examines the fast and efficient calculation method of compact FDs employing a recursive filtering algorithm. The compact FD-TD methods are a low-dispersion scheme. By recursive filtering algorithm we can implement the higher order compact FD-TD analysis as a simple code.

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