Transverse Confinement of Elastic Waves near the Expected Localization Transition in a Highly Porous Glass Network

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1. Introduction

Given the recent successful experimental observation of the Anderson localization of elastic waves in a three-dimensional network of aluminum beads [1], the investigation of wave transport in similar systems is of considerable interest. Here, we use ultrasonic techniques to study departures from normal diffusive behavior in a highly porous network, with a solid volume fraction of only 0.33, formed from sintered glass beads.

At intermediate frequencies, where the wavelength is comparable with the pore sizes in the medium, very strong scattering is observed, with values of longitudinal wave vector times mean free path kl_s close to 1. Previous measurements in this system revealed plateaus in the diffusion coefficient and density of states over a wide range of frequencies [2, 3] in the strong scattering regime.

To look for possible deviations from normal diffusive transport in this system, the transverse spreading of a tightly focused input beam was measured on the opposite face of the sample.

2. Materials and Methods

Experiments were performed on a highly porous, disordered sintered glass bead network. The material was constructed by sintering a 1:1 mixture of polydisperse glass and iron beads with diameter of about 120 μm. The iron was removed by etching after sintering, leaving a highly porous, disordered structure. The resulting material had a glass volume fraction of $\phi = 0.33$ Samples with thickness L of 2.37 and 3.69 mm were investigated. Pulsed ultrasonic measurements were performed using a point source geometry depicted in **Fig. 1**. The transmitted field was measured using a hydrophone in frequency range from 1 to 6 MHz.
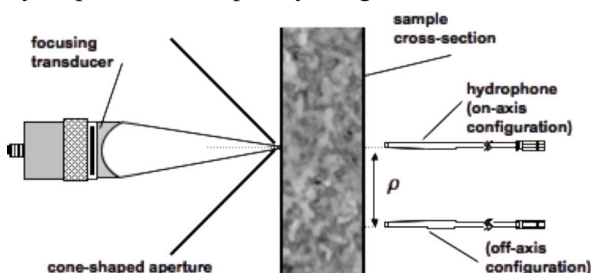

Fig. 1   Experimental set-up.

With the sample fixed in position, the spatial profile of the transmitted field was measured by scanning the hydrophone over a 125 by 125 grid with a step length of 0.2 mm. To make measurements with better statistics at several transverse distances ρ, the hydrophone was scanned along a line from -14 mm to + 14 mm for more than 2000 positions of the sample relative to source and detector.

3. Results and Discussion

The transmitted intensity $I(\rho)$ through the thicker sample ($L = 3.69$ mm) was determined at each point from the magnitude squared of the FFT of the measured field. **Fig. 2** shows the logarithm of the average intensity profile at 4.5 MHz, averaged over 101 frequencies between 4.45 and 4.55 MHz. The source is located at $\rho = 0$. The intensity profile may be characterized by a 2D Gaussian function and the best fit gives the static transverse width $w = 3.6$ mm, or $0.98L$. This width includes a contribution from absorption, making it difficult to relate it to possible confinement of waves associated with localization. In order to eliminate the effect of absorption, the time-dependent transmitted intensities $I(\rho,t)$ were
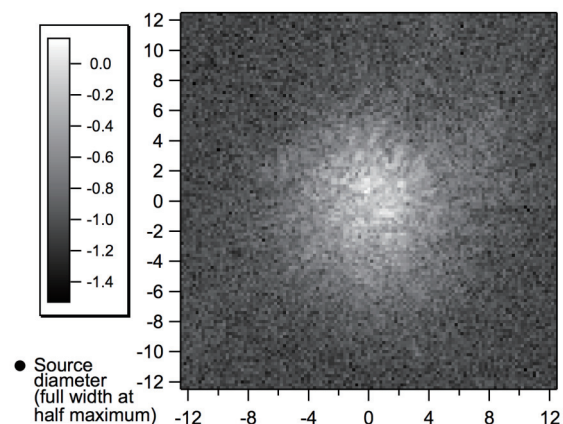

Fig. 2   Near-field static intensity profile at 4.5 MHz.

digitally filtered with a 5% bandwidth around each central frequency and averaged for the same ρ. From these measurements, the dynamic intensity ratio $I(\rho,t)/I(0,t)$ can be determined. The dependence of $I(\rho,t)/I(0,t)$ on ρ at six different times is shown in **Fig. 3**. As discussed by Hu *et al*.

[1], absorption that has been a major problem in experimental confirmation of Anderson localization in a three-dimensional system, cancels out in the ratio $I(\rho,t)/I(0,t)$.
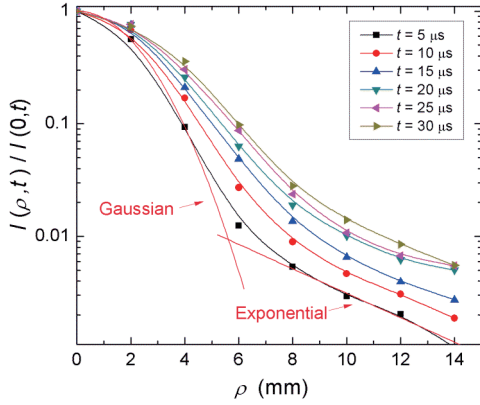

Fig. 3   Dynamic intensity ratio versus transverse distance ρ at 4.5 MHz.

The spatial intensity profile is Gaussian if the waves propagate in diffusive way.   In our case, behavior is very different – the intensity profile is non-Gaussian with a broad tail for large ρ that cannot be explained by ordinary diffusion.   The transverse spreading also slows down at long times.
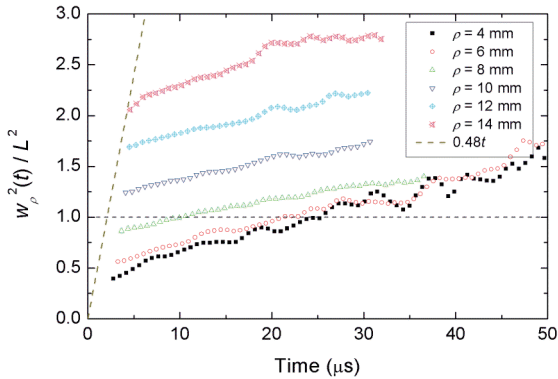

Fig. 4   Transverse width at 4.5 MHz vs time for different ρ.

This transverse spreading may be characterized the dynamic transverse width $w_\rho(t)$, defined as $w_\rho^2(t) = -\rho^2 \big/ \ln\big(I(\rho,t)/I(0,t)\big)$.   **Fig. 4** shows the time dependence of the transverse width at 4.5 MHz for different ρ.   The dashed line represents the linear time dependence of $w_\rho^2(t)$ that would be expected for diffusive waves. This figure clearly shows that the transverse width increases with time more slowly than for diffuse waves. However, $w_\rho(t)$ still increases with time at long times and does not fully saturate for all frequencies studied in the same way as was found for samples made from aluminium beads [1].   These data indicate that the

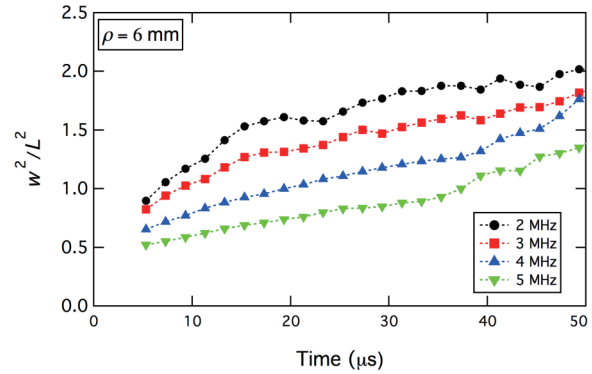behavior is subdiffusive in this frequency range for the glass samples.

Fig. 5   Time dependence of the dynamic transverse width at different frequency.

We also find that the width decreases with frequency, as seen in **Fig. 5** for a transverse distance ρ = 6 mm.   This indicates that the departure from normal diffusive behavior is greater at higher frequencies. Thus, these transverse confinement measurements indicate that sound waves are not localized but are approaching localization as the frequency is increased.

4. Conclusions

The propagation of ultrasonic waves through a highly porous, disordered sintered glass bead system was investigated. The mean square width of the transverse intensity profile was found to increase with propagation time more slowly than expected for diffuse waves, a signature of renormalization of the diffusion coefficient due to interference effects that can lead to Anderson localization.   As the frequency is increased, the long-time transverse width decreases but remains larger than the sample thickness, indicating that, over the range of frequencies studied, a transition to localized modes is being approached but not yet reached.   Additional experiments at higher frequency are needed to investigate Anderson localization in such systems.

References
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