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Basic Investigation on Acoustic Imaging Technique using Temporal Capon Algorithm and Filtered Backprojection Algorithm

時間 Capon 法とフィルタ補正逆投影法による音響イメージング手法に関する基礎検討

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1. Introduction

In this paper, we propose a new acoustic imaging technique and show its simulation results. Our method extends the Temporal Capon (T-Capon) and integrates with the Filtered Backprojection (FBP) technique[1)]. The proposed method uses FBP in static sensor settings, which is different from its conventional usage such as computer tomography that rotates sensors.

2. Proposed method

2.1 Temporal Capon Algorithm

T-Capon is a method to estimate time distributions of signal powers. According to the normal Capon algorithm[2)], we can describe the signal power $g(t)$ as follows:

$$g(t) = 1/\{2\boldsymbol{a}(t)^H \boldsymbol{R}_{xx} \boldsymbol{a}(t)\}$$

$$\begin{cases} \boldsymbol{R}_{xx} = E[\boldsymbol{X}\boldsymbol{X}^H] \ \ (\boldsymbol{X} = [x_1, x_2, \cdots, x_K]^T) \\ \boldsymbol{a}(t) = [\exp(2\pi f_1 t), \exp(2\pi f_2 t), \cdots, \exp(2\pi f_K t)]^T \end{cases}$$

where $\boldsymbol{a}(t)$ is the mode vector, $\boldsymbol{R}_{xx}$ is the correlation matrix of receiving signal, $x_n \ (n = 1,2,\cdots,K)$ is the receiving signal of frequency f_n, $\mathrm{E}[\cdot]$ represents the statistical average operation, and the superscript $(\cdot)^H$ and $(\cdot)^T$ denotes the Hermitian operation and the matrix transpose operation, respectively. We define the sound field in the polar coordinate as $f(u,r)$, where r is the distance from the sensor, u is defined as $\sin\theta$ and θ is an angle. Here, the following equation holds.

$$g(t) = \int_{-1}^{1} f(u, tc) du \qquad (1)$$

where c is the sound velocity.

2.2 Extension of Temporal Capon Algorithm

In our method, we assume that the receiving signals are plane waves and use an array sensor arranged on the x-axis. Let us consider two sensors A at $x = 0$ and B at $x = x_1$. The path length of a wave that arrives at the sensor A is $x_1 \sin\theta / c (= x_1 u / c)$ longer than that arriving at sensor B as shown in **Fig.1**. About the path length at sensor x, the following equation must be fulfilled.

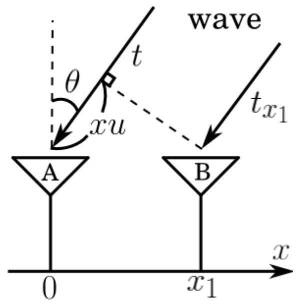

Fig.1 Relation of two sensors

$$t_x = t - xu/c$$

where t and t_x are the receiving times at sensor $x = 0$ and $x = x$. According to equation (1), the result of T-Capon at a sensor x can be described as,

$$g(t_x) = \int_\gamma f(u,r) ds \qquad (2)$$

$$(\gamma : r = xu + ct_x, -1 < u < 1)$$

Next, we introduce a new variable φ as shown in **Fig.2**. φ satisfies the following equation:

$$\varphi = \tan^{-1} x + \pi/2$$

Because φ is a function of x, we can describe the result of T-Capon at a sensor x as $g(t_x, \varphi)$. In this case, equation (2) becomes,

$$g(t_x, \varphi) = \int_\gamma f(u,r) ds$$

$$= \iint_{-\infty}^{\infty} f(u,r)\delta(xu + ct_x - r) du dr \qquad (3)$$

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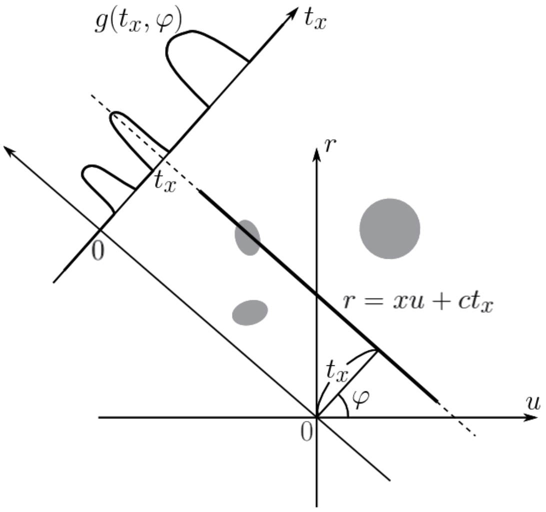

Fig.2 Radon transform

Equation (3) is a mathematical expression of the Radon transfer. Hence we can apply FBP to the expression.

2.3 Filtered Backprojection Algorithm

The two-dimensional inverse Fourier transform of $f(u,r)$ is described as

$$f(u,r) = \iint_{-\infty}^{\infty} F(f_u, f_r) \exp\{i2\pi(f_u u + f_r r)\} df_u df_r \quad (4)$$

where $F(f_u, f_r)$ is the two-dimensional Fourier transform of $f(u,r)$. According to the Projection theorem, we can describe the one-dimensional Fourier transform of $g(t_x,\varphi)$ by t_x as follows:

$$G_\varphi(\omega) = F(\omega\cos\varphi, \omega\sin\varphi) \quad (5)$$

Using (5) and **Fig.2**, equation (4) becomes,

$$f(u,r) = \int_0^\pi \int_{-\infty}^{\infty} G_\varphi(\omega) \exp\{i2\pi\omega(u\cos\varphi + r\sin\varphi)\} |\omega| d\omega d\varphi = \int_0^\pi \{\int_{-\infty}^{\infty} |\omega| G_\varphi(\omega) \exp(i2\pi\omega t_x) d\omega\} d\varphi \quad (6)$$

We define a filter function $h(t_x)$ as follows:

$$h(t_x) = \int_{-\infty}^{\infty} |\omega| \exp(i2\pi\omega t_x) d\omega \quad (7)$$

Using (7), equation (6) becomes,

$$f(u,r) = \int_0^\pi g(t_x,\varphi) * h(t_x) d\varphi \quad (8)$$

By using equation (8), we can estimate the sound field $f(u,r)$.

However, the value of φ is discrete in actual situations because the value of φ depends on that of x. Therefore, when we use a linear sensor array, $f(r,u)$ is expressed as follows:

$$f(u,r) = \sum_{m=0}^{M} \{g(t_x, \varphi(m)) * h(t_x)\}$$

$$(\varphi(m) = \tan^{-1}(m\Delta x) + \pi/2)$$

where Δx is the distance between adjoining sensors and M is the number of sensors.

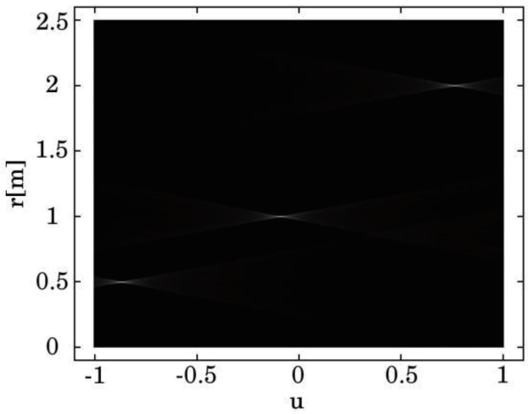

Fig.3 Simulation result

3 Simulation result

We conducted simulations using the proposed method. In the simulation three point sources were placed at $(r,u) = \{(0.5, -0.87), (1, -0.087), (2, 0.77)\}$ (the unit of r is meter). SNR is 20 dB. A multicarrier signal consisting of 41 subcarriers whose frequencies range from 34 to 39 kHz (125 Hz interval) were used. 64 receiver sensors were arranged at intervals of 10 mm. The result of the simulation is shown in **Fig.3**.

4 Conclusion and Future

We proposed a new method for acoustic imaging. We conducted simulations using the method and obtained images. However, as shown in **Fig.3**, this image is not clear. It may be because the value of φ do not occupy the range $[0, \pi]$. One of our future works is to solve this problem.

References

1) G.N.Ramachandran and A.V.Lakshminarayan: Proc.Natl.Acad.Sci.U.S.A. **68** (1971) 2236.

2) J. Capon: IEEE Proc. **57** (1969) 1408.