A simulation of wave propagation of Bone-conducted sound using Viscoelastic equation

粘弾性方程式を用いた骨導音の伝搬シミュレーション

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1. Introduction

Bone-conducted sounds can be perceived even above the audible frequency range. In addition, the mechanism of bone-conducted perception ultrasound (BCU) seems to be different from that of sounds audible because some profoundly hearing-impaired people as well as those with normal hearing can hear some sounds when BCU is present [1]. The unexplained perception mechanism of BCU has been studied from both psychological and biological perspectives. The cochlea is a key factor in that mechanism, because activations of the auditory cortex [2] and the auditory brainstem [3] were observed.

Wave propagation in the inner head has been estimated by using a finite difference time domain (FDTF) method with linear acoustic and elastic equations in order to understand the physical phenomena of BCU [4]. The viscosity attenuation of bio-tissues has much effect on the wave propagation of the bone-conducted sound. however, that was not considered in the foregoing study. We previously implemented an FDTD analysis with a viscoelastic constitutive equation in order to obtain a more accurate simulation of BCU wave propagation [5]. Meanwhile, the necessity and the reliability of the proposed method were not addressed.

In this paper, the reliability of the viscoelastic simulation is evaluated by comparing the numerical analysis and an actual measurement using a simple experimental model. Also the correction handling with a propagation speed in the viscoelastic simulation was introduced.

2. Actual measurement and numerical simulation with simple model

2.1. Experimental setup for actual measurement

The experimental setup for the actual measurement is shown in **Fig. 1**. Silicon rubber $(60 \times 60 \times 5 \text{ mm})$ was placed vertically between a transmitter and receiver. The output signal from a function generator was a one-period sine wave of 5 Vpp. The signal was applied to the transmitter after a 10-dB gain by a power amplifier. The 4096 received signals were averaged.

2.2. Method of numerical analysis

The FDTD with viscoelastic constitutive equation was used to calculate the wave propagation in the simulation model. Inelasticity of media is normally described using quality factor Q. The viscoelastic constitutive equation, which is a convolutional style, is difficult to implement in a time domain. It can be easily calculated, however, using a recently proposed method that uses an innovative memory variable to avoid convolution [6].

The viscoelastic constitutive equation can be formulated by following equations (1) to (4) and Newton's second law.

$$\frac{\partial \sigma_{ii}}{\partial t} = \left\{ \left(\lambda + 2\mu \right) \left[1 - \sum_{l=1}^{L} \left(1 - \frac{\tau_{el}^{P}}{\tau_{ol}} \right) \right] - 2\mu \left[1 - \sum_{l=1}^{L} \left(1 - \frac{\tau_{el}^{S}}{\tau_{ol}} \right) \right] \right\} \nabla \cdot \mathbf{v} + 2\mu \left[1 - \sum_{l=1}^{L} \left(1 - \frac{\tau_{el}^{S}}{\tau_{ol}} \right) \right] \frac{\partial v_{i}}{\partial i} + \sum_{l=1}^{L} R_{iil}$$
(1)

$$\frac{\partial \sigma_{ij}}{\partial t} = \left\{ \mu \left[1 - \sum_{l=1}^{L} \left(1 - \frac{\tau_{el}^{S}}{\tau_{ol}} \right) \right] \right\} \left(\frac{\partial v_{j}}{\partial i} + \frac{\partial v_{i}}{\partial j} \right) + \sum_{l=1}^{L} R_{ijl}$$
(2)

Here, σ is a strain tensor, v is a particle velocity vector, and λ and μ indicate Lame's constants. The *L*th-order relaxation mechanism is achieved using memory variable *R* because the viscoelastic medium depends on the history of the strain tensor.

$$\frac{\partial R_{iil}}{\partial t} = \left\{ \frac{1}{\tau_{\sigma l}} \left(\lambda + 2\mu \right) \left(1 - \frac{\tau_{el}^P}{\tau_{\sigma l}} \right) - R_{iil} - 2\mu \left(1 - \frac{\tau_{el}^S}{\tau_{\sigma l}} \right) \right\} \nabla \cdot \mathbf{v} + 2\mu \left(1 - \frac{\tau_{el}^S}{\tau_{\sigma l}} \right) \frac{\partial v_i}{\partial i}$$
(3)

$$\frac{\partial R_{ijl}}{\partial t} = \frac{1}{\tau_{\sigma l}} \left\{ \mu \left(1 - \frac{\tau_{cl}^S}{\tau_{\sigma l}} \right) - R_{ijl} \right\} \left(\frac{\partial v_i}{\partial j} + \frac{\partial v_j}{\partial i} \right)$$
(4)

Here, τ_{sl} and $\tau_{\sigma l}$ indicate the stress and strain relaxation times of the *l*th mechanism. In this simulation, a third-order relaxation mechanism (*L*=3) was used.

The box bordered in white in Fig. 1 indicates the analyzed area, which was 30×33 mm. It consisted of 1499×1650 grids with a grid space of 20μ m.

The longitudinal sound speeds (c_p) of the water and silicon rubber were set to 1,477 and 947 m/s, and the transverse ones (c_s) were set to 0 and 548 m/s, respectively. Numerical calculations were done in cases where Q was equal to 30, 50, 100, 200, and 500.

2.3. Results

Figure 2 plots the peak-to-peak values of the received signal as a function of frequency in each condition when the one-period cosine wave with the Hanning window was applied. The viscoelastic simulation with a Q value of 100 shows closer correspondence to the actual measurement than the elastic simulation.

3. Correction of propagation speed in viscoelastic simulation

The attenuation behavior of the silicon rubber could be reasonably represented by using the viscoelastic constitutive equation as mentioned above. The estimated propagation speed of media, however, has increased as the quality factor Q decreases in the viscoelastic simulation. The change of the propagation speeds in each Q is shown as the blue broken line in Fig. 3.

To improve this issue, we propose the correction handling for the propagation speed by dividing each speed ratio as following;

 $c_{ratio}(Q) = c_{viscoelastic}(Q) / c_{elastic}(Q).$ (5) Consequently, the red solid line in Fig. 3 was

calculated the changed propagation speed became almost constant.

4. Conclusion

In the simple model, the viscoelastic simulation was more consistent with the actual measurement than the elastic simulation was. Also the issue that sound speed in the viscoelastic simulation was dependent of the quality factor Q was improved. While the value of Q needs to be examined in more detail, however, the obtained results indicated that more accurate calculation can be carried out by using the viscoelastic simulation method.

References

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Fig. 1 Experimental setup for the actual measurement. The box bordered in white indicates the analytic area in the numerical simulation.



Fig. 2 Comparison of amplitudes of the observed and calculated waveforms.



Fig. 3 The comparison of the ratio of propagation speed about the correction handling.