# Equivalent orthorhombic mm2 symmetry material constants of 2-2 mode piezocomposites for ultrasonic transducers 

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## 1. Introduction

Piezocomposites are made by combining a piezoelectric ceramic with a polymer. These piezocomposites are classified according to the connectivity of the two phases. In this study, we derived the equivalent properties of the 2-2 mode piezocomposites such as the elastic, piezoelectric and dielectric constants. These constants were obtained by means of asymptotic averaging method [1,2]. Validity of the derived material constants was verified by comparing the resonant spectra from the FEM models with the equivalent properties and those models with a full 2-2 composite structure. The methodology of deriving the equivalent properties of a 2-2 piezo-composite in this study will be useful for designing 2-2 piezocomposites and analyzing its performance for various transducers.

## 2. Theory

Axes, directions and dimensions are defined in this paper as in Fig. 1. We analyzed the electro-mechanical impedance spectra of various piezocomposite samples both with analytic equations and with FEM. $\mathrm{k}_{\text {eff }}$ can be shown as eq. 1,

$$
\begin{equation*}
k_{e f f}=\sqrt{\frac{f_{a}^{2}-f_{r}^{2}}{f_{a}^{2}}} \tag{1}
\end{equation*}
$$

In the equation, $f_{\mathrm{a}}, f_{\mathrm{r}}$ are anti resonance frequency, resonance frequency respectively. Properties of the materials for the composite are shown in Table I. C, $e, \varepsilon$ are components of stiffness, piezoelectric, dielectric matrix respectively, and Q is mechanical quality factor. $V_{1}$ is longitudinal sound velocity, $V_{\mathrm{s}}$ is shear sound velocity. $\alpha_{1}$, is longitudinal attenuation coefficient, $\alpha_{\mathrm{s}}$ is shear attenuation coefficient at $3.5 \mathrm{MHz} . \rho$ is density.

The equations to get each component of stiffness, piezo-electric and dielectric matrix is shown in eq. 2. In the equations, the indexes $\mathrm{C}, \mathrm{P}$ show the ceramic, polymer phases respectively, $v$ is the ceramic volume fraction, and upper bar means the composite phase.

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$$
\begin{align*}
& \overline{c_{22}}=\left\langle c_{22}{ }^{-1}\right\rangle^{-1}, \overline{c_{12}}=\left\langle c_{12} / c_{22}\right\rangle \overline{c_{22}}, \\
& \overline{c_{23}}=\left\langle c_{23} / c_{22}\right\rangle \overline{c_{22}} \text {, } \\
& \overline{c_{11}}={\overline{c_{12}}}^{2} / \overline{c_{22}}+\left\langle c_{11}-c_{12}{ }^{2} / c_{22}\right\rangle, \\
& \overline{c_{13}}=\overline{c_{12}} \overline{c_{23}} / \overline{c_{22}}+\left\langle\mathcal{c}_{13}-\mathcal{c}_{12} c_{23} / c_{22}\right\rangle, \\
& \overline{c_{33}}={\overline{c_{23}}}^{2} / \overline{c_{22}}+\left\langle c_{33}-c_{23}{ }^{2} / c_{22}\right\rangle \text {, } \\
& \overline{c_{44}}=Y_{1} / Z, \overline{c_{55}}=\left\langle c_{55}\right\rangle, \overline{c_{66}}=\left\langle c_{66}{ }^{-1}\right\rangle^{-1}, \\
& \overline{e_{15}}=\left\langle e_{15}\right\rangle, \overline{e_{24}}=Y_{2} / Z, \overline{e_{32}}=\left\langle e_{32} / c_{22}\right\rangle / \overline{c_{22}}, \\
& \overline{e_{31}}=\overline{e_{32}} \bar{c} \bar{c} / \bar{c} / \overline{c 2}+\left\langle e_{31}-e_{32} c_{12} / c_{22}\right\rangle, \\
& \overline{e_{33}}=\overline{e_{32}} \overline{c_{23}} / \overline{c_{22}}+\left\langle e_{33}-e_{32} c_{23} / c_{22}\right\rangle, \\
& \overline{\varepsilon_{11}}=\left\langle\varepsilon_{11}\right\rangle, \overline{\varepsilon_{22}}=Y_{3} / Z, \\
& \text { and } \overline{\varepsilon_{33}}=-e_{32}{ }^{2} / \overline{c_{22}}+\left\langle\varepsilon_{33}+e_{32}{ }^{2} / c_{22}\right\rangle \text {, } \\
& \text { where, } \\
& X=c_{44} \varepsilon_{22}+e_{24}{ }^{2}, Y_{1}=\left\langle c_{44} / X\right\rangle, Y_{2}=\left\langle e_{24} / X\right\rangle \text {, } \\
& Y_{3}=\left\langle\varepsilon_{22} / X\right\rangle, \mathrm{Z}=Y_{2}^{2}+Y_{1} Y_{3} \text { and }  \tag{2}\\
& <(.)\rangle=v(.)^{\mathrm{C}}+(1-v)(.)^{\mathrm{P}}
\end{align*}
$$
\]

## 3. Verification and modification of equivalent properties

We analyzed the electromechanical impedance spectra of several resonance models that show the effect of each material constant. LTE1 is Length Thickness Extensional mode through the 1 direction. LTE2 is Length Thickness Extensional mode through the 2 direction. TE is Thickness Extensional mode through the 3 direction. LE is Length Extensional mode through the 3 direction. TS1 is Thickness Shear mode through the 3 direction. TS2 is Thickness Shear mode through the 3 direction. Dimensions and the directions of polarization and electric field of each model are summarized in Table II.

We compared the impedance spectra of full 2-2 piezocomposite FEM models and with those of the models with the equivalent properties. The highest difference was observed at the TE model (Fig. 2). The material constants most influential on each resonance as shown in Table II were modified to fit the full model data, and the fitted results in Fig. 3 matched each other very well. The final fitted properties are shown in Table III.

## 4. Conclusions

The properties of a piezoelectric material having its elastic, dielectric, and piezoelectric constants equivalent to those of a 2-2 piezocomposite were derived using the asymptotic averaging method. The derived constants was confirmed valid by comparing the impedance spectra of the equivalent material with those of full 2-2 mode piezocomposite structures.

## References

1. R. R. Ramos, J. A. O. Hernandez, J. B. Castillero, and F. J. Sabina, Mechanics of Composite Materials, 32 (1996) 286-291.
2. K. Y. Hashimoto, M. Yamaguchi, Proceedings of the IEEE Ultrasonics Symposium (1986) 697-702.

Fig. 1 Schematic of 2-2 piezo-composite.

Table III. Equivalent properties of 2-2 Piezo-composite

| 2-2 piezo-composite |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $C_{11}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 6.64 | $e_{31}\left(\mathrm{C} / \mathrm{m}^{2}\right)$ | -3.14 |
| $C_{22}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 2.50 | $e_{32}\left(\mathrm{C} / \mathrm{m}^{2}\right)$ | -1.14 |
| $C_{33}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 5.17 | $e_{33}\left(\mathrm{C} / \mathrm{m}^{2}\right)$ | 18.94 |
| $C_{12}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 1.51 | $e_{15}\left(\mathrm{C} / \mathrm{m}^{2}\right)$ | 11.24 |
| $C_{13}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 3.11 | $e_{24}\left(\mathrm{C} / \mathrm{m}^{2}\right)$ | 0.01 |
| $C_{23}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 1.51 | $\varepsilon_{11}{ }^{\mathrm{S}} / \varepsilon_{0}$ | 915.23 |
| $C_{44}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 0.48 | $\varepsilon_{22} \varepsilon_{0}$ | 13.02 |
| $C_{55}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 1.61 | $\varepsilon_{33} / \varepsilon_{0}$ | 841.06 |
| $C_{66}{ }^{\mathrm{E}}$ | $\left(10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)$ | 0.47 | $\rho\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ | 5840 |
|  |  |  | $Q_{\mathrm{m}}$ | 30 |



Fig. 2 Electro-mechanical impedance spectra of TE.


Fig. 3 (a) LTE1 (b) LTE2 (c) TE (d) LE (e) TS1 (f) TS2.


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