# Sound Propagation on Circular Concave Surface De－ pending on Radius of Boundary． 

円形凹境界に沿った音波の伝搬特性の境界半径に対する依存性

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## 1．Introduction

The phenomenon of＂whispering gallery＂ which observed in the circular hall is that a voice spoken near $t$ a circular concave wall propagate on the wall，and we can clearly hear the voice even if the observer is far from the speaker and the voice is a whisper．In the end of 19th century，J．W．Strutt （Lord Rayleigh）investigated these phenomenon， and elucidated＂whispering gallery（WG）modes＂ which was restrained to the vicinity of the concave surface in the low－order mode［1］．

Recent researches of acoustic WG modes were to derive the sound pressure distribution ana－ lytically，by math calculation，were to facilitate the physical interpretation of sound pressure distribu－ tion，and so on．However，most of the research were to study steady－state response［2－4］．Then，the prop－ erty of acoustic WG modes in the finite length sound is not well investigated，thus the change of WG modes by sound source positions，frequencies， radius of boundary and signal lengths have not been clear．

In this paper，we analyze the sound propaga－ tion on circular concave surface depending on radi－ us of boundary by Transmission－Line Matrix （TLM）method［5］．The TLM method is based on the Huygens＇principle，which gives the time do－ main solution of the wave field．Circular concave surface is the simplest concave．We investigate the change of sound propagation region corresponding to radius of boundary．The results of this analyzes are expected to be applied to measurement of phys－ ical properties on the vicinity of the concave sur－ face．

## 2．Condition to Simulate

Figure 1 shows the simulated sound field surrounded by circular concave surface．The me－ dium in which sound propagates is air，and concave surface is rigid wall．Sound frequency is 10 kHz ， wavelength $\lambda$ is 0.0337 m ．Radius of boundary $r_{0}$

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Fig． 1 Simulation sound of surrounded by circular concave surface．


Fig． 2 Distribution of normalized time－averaged power at $d=$ zero and 20 cycle signal；（a）$r_{0}=6 \lambda$ ，（b） $8 \lambda$ ，（c） $10 \lambda$ ，（d） $12 \lambda$ ，（e） $14 \lambda$ ，（f） $16 \lambda$
and distance of sound source from boundary $d$ are changed from $6 \lambda$ to $16 \lambda$ ，from zero to $r_{0}-2$ $\lambda$ ，respectively．Signal length is 20 cycles．The length of line sound source $s_{\mathrm{L}}$ is 0.06 m ，and line sound source is located on $r_{0}-\left(d+s_{\mathrm{L}}\right)<r<r_{0}-d$ ，$\theta$ $=180 \mathrm{deg}$ ．Sound waves are outputted the -90 deg


## Normalized power

Fig. 3 Distribution of normalized time-averaged power at $d=3 \lambda$ and 20 cycles signal; (a) $r_{0}=6 \lambda$, (b) $8 \lambda$, (c) $10 \lambda$, (d) $12 \lambda$, (e) $14 \lambda$, (f) $16 \lambda$
direction from $x$-axis. Calculation time $t_{\mathrm{c}}$ is set the time required to propagate the sound on the surface about 5 laps.

## 3. Results and Discussion

Figure 2 shows the distributions of normalized time-averaged power at $d=0$ and 20 cycles signal, $r_{0}$ are changed from $6 \lambda$ to $16 \lambda$. Time-averaged power $P(r, \theta)$ define as follows,

$$
\begin{equation*}
P(r, \theta)=\frac{1}{t_{\mathrm{c}}} \sum_{t=0}^{t_{\mathrm{c}}}(R(r, \theta, t))^{2} \tag{1}
\end{equation*}
$$

where $R(r, \theta, t)$ is received sound at position $(r, \theta)$ and time $t$. In all cases of Fig. 2, sound propagates on circular concave surface and the distributions are almost identical. $P(r, \theta)$ is distributed within the range of $r_{0}-\lambda \leq r \leq r_{0}$ and exhibits a maximum at the surface, regardless of $r_{0}$. Furthermore, $P(r, \theta)$ fluctuates in the radial direction periodically. It is thought that this result is a kind of WG modes.

Distributions of normalized time-averaged power at $d=3 \lambda$ and 20 cycles signal are shown in Fig. 3; $r_{0}$ are changed from $6 \lambda$ to $16 \lambda$. At $r_{0}=8 \lambda$ and $16 \lambda$, the distributions on $r_{0}-\left(d+s_{\mathrm{L}}\right)<r<r_{0}-d$ are shaped like a triangle and square, respectively. These distributions occur due to reflection sounds a regular polygon path. $P(r, \theta)$ exhibits a maximum at $r_{0}-d-\lambda<r<r_{0}-d$, independently of $r_{0}$.

In Fig. 2 and 3, $P(r, \theta)$ is large within the range ( $r_{0}-d-\lambda<r<r_{0}-d$ ) and exhibits a maximum there, independently of $r_{0}$ and $d$. Therefore, it is supposed that similar distribution can be reproduced when the length of sound source is about wavelength.

Figure 4 shows distribution of normalized time-averaged power at 20 cycles and $d=0.5 r_{0}$, and $r_{0}$ are changed from $6 \lambda$ to $16 \lambda$. At $10 \lambda \leq r_{0} \leq 16 \lambda$,


Fig. 4 Distribution of normalized time-averaged power at 20 cycle signal and $d=0.5 r_{0}$; (a) $r_{0}=6 \lambda$, (b) $8 \lambda$, (c) $10 \lambda$, (d) $12 \lambda$, (e) $14 \lambda$, (f) $16 \lambda$
normalized time-averaged power distributions do not change. On the other hand, at $r_{0}=6 \lambda$ and $8 \lambda$, these distributions change. However, this change is due to the length of the sound source. Thus, the ratio between $r_{0}$ and $d$ is closely related to the region of sound propagation.

## 4. Conclusion

We analyzed sound propagation on circular concave surface depending on radius of boundary using the sound propagation simulation by TLM method. As the results, time-averaged power $P(r, \theta)$ shows the periodic radially fluctuation at $d=0$ : thus, it is supposed the formation of WG modes. $P(r, \theta)$ is large within the range $\left(r_{0}-d-\lambda<r<r_{0}-d\right)$ and takes a peak there, regardless of $r_{0}$ and $d$. Consequently, it is thought that similar distribution can be reproduced if the length of sound source is about wavelength. Furthermore, the ratio $r_{0}$ and $d$ is more important than the value of both $r_{0}$ and $d$.

In the future, the investigation of received sound is planned. It contains important information about the formation of WG modes to received sounds. Experimental verification is also planned. We need to verify the simulation results and confirm the actual sound pressure distribution.

## References

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