A Dynamic Analysis of Ultrasonically Levitated Droplet with Moving Particle Semi-implicit and Distributed Point Source Method

粒子法と分布点音源法による超音波浮揚液滴の 動的シミュレーション

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1. Introduction

Ultrasonic levitation has recently been drawing attention as a way of non-contact transportation of small objects, such as liquid droplets, in bioengineering and manufacturing industry. The small objects in the finite amplitude sound field have been known to be trapped near the pressure node of the standing wave with the effect of acoustic radiation force [1-2]. Many experimental reports [3] are presented related to the droplet levitation and their shape in the air. The droplet with large volume is reported to turn its shape from sphere to spheroid when they are exposed in the intense sound pressure field. Several analytical reports have mentioned this phenomenon [4-6], however, almost no report carried out dynamical simulation on the shape change of the droplet. In this paper, the levitated droplet shape is simulated by coupling two gridless analysis methods, the one is distributed point source method (DPSM, [7]) and the other is moving particle semi-implicit (MPS, [8]) method.

2. Calculation Procedure

Acoustic radiation stress Π and pressure P_{rad} are expressed by the sound pressure p_a and particle velocity u_a , which is calculated by acoustic analysis performed in DPSM as

$$\Pi = P_{rad} \mathbf{I} + \rho_0 \left\langle \mathbf{u}_a^t \mathbf{u}_a \right\rangle$$

$$P_{rad} = \frac{\left\langle p_a^2 \right\rangle}{2\rho c^2} + \rho_0 \frac{\left\langle \mathbf{u}_a^2 \right\rangle}{2}, \qquad (1)$$

$$p_a = \frac{j\omega\rho}{2\pi} \iint V_0 \frac{e^{-jkr}}{r} dS, \quad \mathbf{u}_a = -\frac{\nabla p_a}{i\omega\rho},$$

where I, ρ_0 , c, ω , k, and V_0 are the unit tensor, density, sound speed, angular frequency, wavenumber in air, and vibration velocity of the transducer. Calculated acoustic radiation pressure is considered in static fluid analysis, which is

Fig. 1 Problem geometry for the ultrasonic droplet levitation





simulated in MPS, as

$$\frac{D\mathbf{U}}{Dt} = -\frac{\nabla P}{\rho_w} - v\nabla^2 \mathbf{U} + g + \frac{\rho_0}{\rho_w} \frac{\partial \langle \mathbf{u}_a^t \mathbf{u}_a \rangle}{\partial \mathbf{n}},$$

$$\nabla^2 P = \frac{\rho_w}{\Delta t} \operatorname{div} \mathbf{U}, \quad P = P_{rad} + \sigma \kappa \text{ (on } \Gamma)$$
(2)

where *P*, *U*, ρ_w , σ , *v*, κ , *g*, *n* and Γ are the static pressure, velocity, density, surface tension, dynamic viscosity of the liquid, curvature of the droplet surface, gravity, normal vector of boundary surface and set of point on boundary.

f = 27.44 kHz $c_{a} = 340 \text{ m/s}$ $\lambda = 12.4 \text{ mm}$ $p_{a} = 1.3 \text{ kg/m3}$ $g = 9.8 \text{ m/s}^{2}$ z = 12.7 mm ransducer $p_{a} = 998 \text{ kg/m}^{3}$ $\sigma_{a} = 72.8 \text{ mN/m}$ $p_{a} = 1.89 \text{ kg/m}^{3}$ $\sigma_{a} = 22.3 \text{ mN/m}$ $reflector \phi 60 \text{ mm}$ x

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Fig. 1 indicates the problem geometry for the ultrasonic droplet levitation. A droplet with initial sphere radius are 0.4 - 1.0 mm is levitated in the sound field, which is generated by the ultrasonic transducer and the reflector. The droplet is initially placed at 2.2 mm above the reflector, and expected to move toward the node of sound field, which is 3.1 mm, 1/4 wavelength, above the reflector.

Fig. 2 shows the sound pressure distribution for the initial time step. The amplitude of the sound pressure is 4 kPa at the antinode of the standing wave. As a result, the radiation pressure on the sound pressure antinode and node are +26 Pa -51 Pa, respectively.

3. Results

Fig. 3 shows the droplet deformation and radiation pressure at the time 0, 2, 5 and 30 ms. The droplet levitates because the vertical positive static pressure balances the gravity, and the shape turns into spheroid because the horizontal negative static pressure balances the surface tension. Figs. 4, 5 show the change in the gravity center position and shape aspect ratio of the droplet. Because of the lower inertia and surface tension, the ethanol droplet changes in the position and aspect ratio are faster than the water droplet. Fig. 6 shows the droplet aspect ratio dependency on surface tension. The numerical result is 10% larger than theory [5], though, they agreed well in tendency.

4. Conclusion

The shape of an ultrasonic levitated droplet was simulated using MPS and DPSM in three dimensional space. The droplet changed its shape from sphere towards spheroid, which agrees well with the shape experimentally known.

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Fig. 3 Ethanol droplet deformation and internal pressure at the time 0, 2, 5, 30 ms with 1mm initial radius.



Fig. 4 Time variation of the gravity center position of the droplet with various initial radii.



Fig. 5 Time variation of droplet aspect ratio with various initial radii.



Fig. 6 Droplet aspect ratio dependency on acoustic Weber number = $\rho_0 a u_a^2 / 2\sigma$.