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Difference of Energy Efficiency between Traveling-wave Type and Standing-wave Type Thermoacoustic Engines
進行波型と定在波型熱音響エンジンのエネルギー効率の違い

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1. Introduction

Heat is converted into acoustic energy in narrow tubes of a stack in a thermoacoustic engine (prime-mover). The thermoacoustic process is characterized by $\omega\tau_\alpha$, where ω is the angular frequnecy of sound and τ_α is the thermal relaxation time defined by Eq. (1) [1].

$$\tau_\alpha = r^2/(2\alpha) \qquad (1)$$

where r is the radius of a narrow tube, and α is the thermal diffusivity of the gas.

The acoustic pressure (P) and the particle velocity (U) of sound at any point are expressed as follows.

$$\mathrm{P} = |P|e^{i(\omega t+\phi)} \qquad (2)$$

$$\mathrm{U} = |U|e^{i(\omega t+\theta)}$$
$$= |U|\cos\Phi e^{i(\omega t+\phi)} + |U|\sin\Phi\, e^{i(\omega t+\phi+\pi/2)} \quad (3)$$

where $\Phi=\theta-\phi$, and i is the unit imaginary number. The real part of the complex numbers (P and U) is the actual physical quantity. For a traveling-wave, there is no phase difference between the particle velocity and the acoustic pressure. For a standing-wave, the phase difference is $\pi/2$. Thus, in Eq. (3), $|U|\cos\Phi$ and $|U|\sin\Phi$ are called the traveling-wave and standing-wave component, respectively[2].

The work flux (I) (acoustic power per unit area of cross section) is expressed as follows[3].

$$\mathrm{I} = 0.5|P||U|\cos\Phi \qquad (4)$$

The energy efficiency (η) of a prime-mover is caclulated as follows.

$$\eta = (I_H - I_C)/q_H \qquad (5)$$

where I_H and I_C are work flux at the higher and lower temperature side of a stack, respectively, and q_H is the heat flux at the higher temperature side of a stack. The upper limit of the energy efficiency is given by the Carnot efficiency (η_{Carnot}).

$$\eta_{Carnot} = 1 - T_C/T_H \qquad (6)$$

where T_H and T_C are the temperature at the higher and lower temperature side of a stack, respectively.

In the present study, numerical simulations of the Rott's equations are performed in oredr to discuss the difference between the traveling-wave and standing-wave type prime-mover.

2. Model

The Rott's equations of momentum and continuity are given as follows[4].

$$dP/dx = -(i\omega\rho\_m)U/(1-\chi\_\nu) \qquad (7)$$

$$dU/dx = -i\omega P[1+(\gamma-1)\,\chi\_\alpha\,]/(\gamma P\_m) + ((dT\_m)/dx)\,U(\chi\_\alpha-\chi\_\nu)/((1-\chi\_\nu)(1-\sigma)\,T\_m) \qquad (8)$$

where x is the position along the narrow tube with its origin at the lower temperature side of a stack, ρ_m, P_m, γ, σ are the mean density, the mean pressure, the ratio of specific heats, and the Prandtl number of the working gas, respectively. χ_α and χ_ν are the thermoacoustic functions[1].

3. Results and Discussions

Numerical simulations of the Rott's equations have revealed that the work flux increases due to the increase of $|U|$ for the traveling-wave type. On the other hand, for the standing-wave type, it is due to the increase of $\cos\Phi$. Furthermore, the traveling-wave and standing-wave components correspond to the increasing rate of $|U|$ and $\cos\Phi$, respectively (Fig. 1). Thus, the traveling-wave type is the amplitude dominant type because the increase of the work flux is due to the increase of the amplitude of U. The standing-wave type is the phase dominant type because the increase of the work flux is due to the increase of the cosine of the phase Φ.

The caculated energy efficiency relative to the Carnot efficiency is shown in Fig. 2 as a function of $\omega\tau_\alpha$ when the radius (r) of a narrow tube of a stack is varied with the fixed temperature difference between the cold and hot ends of a stack (T_C=291 K and T_H=600 K). The optimal condition is $\omega\tau_\alpha$=0.12 and 1.5 for the traveling-wave and the standing-wave type, respectively. It nearly agrees with the theoretical prediction by Tominaga[5] and Inoue[6] that it is $\omega\tau_\alpha \ll 1$ and $\omega\tau_\alpha$=3 for traveling-wave and standing-wave type, respectively. While the energy efficiency at the optimal condition for the traveling-wave type is higher than that for the standing-wave type, the

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energy efficiency for the standing-wave type is higher than that for the traveling-wave type at relatively higher $\omega\tau_\alpha$.

For the traveling-wave type, the energy efficiency decreases for very small $\omega\tau_\alpha$ because of the decrase of $|P|$ due to the viscous damping. For the standing-wave type, on the other hand, it decreases for relatively small $\omega\tau_\alpha$ because the increase of cosΦ is suppressed as the phase shift of U is suppressed due to the faster heat exchange between the wall and a gas parcel.

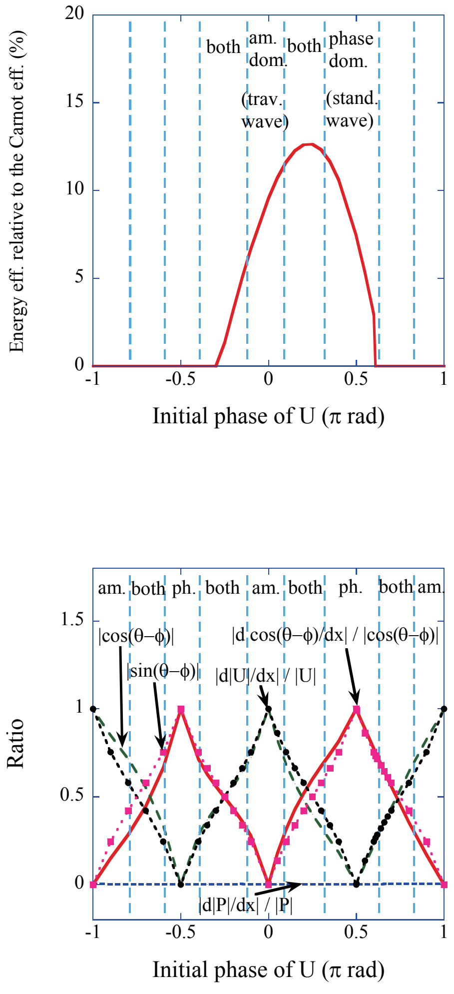

Fig. 1 The result of the numerical simulation on the energy efficiency relative to the Carnot efficiency (above) and the ratio of the traveling-wave and standing-wave components as well as the ratio of the increasing rate of each component of the work flux (below) as a function of the initial phase difference (Φ) between the particle velocity and the acoustic pressure (41 Hz).

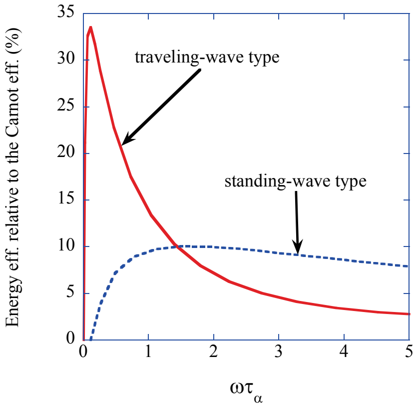

Fig. 2 The result of the numerical simulation on the energy efficiency relative to the Carnot efficiency as a function of $\omega\tau_\alpha$ for traveling-wave and standing-wave type with T_H=600 K and T_C=291 K (41 Hz).

4. Conclusions

Numerical simulations of the Rott's equations have revealed that the mechansim of the dependence of the energy efficiency on $\omega\tau_\alpha$ is different between the traveling-wave and standing-wave type engine because the mechansim of the increase in the work flux is completely different.

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