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Proceedings of Symposium on Ultrasonic Electronics, Vol. 35 (2014) pp. 425-426
3-5 December, 2014

Analysis of leaky guided waves propagating in a water-filled pipe
液体を満たしたパイプを伝搬する漏えいガイド波の解析

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1. Introduction

Cylindrical pipes are widely used in industries such as nuclear power plants and micro total analysis systems (µTAS). Nondestructive evaluation (NDE) of such pipes is therefore crucial. NDE as well as ultrasonic flowmeters can be used to characterize pipes filled with fluid. Guide wave of a hollow pipe was investigated theoretically by Gazis[1], and we previously expanded on the theory proposed by Gazis for a fluid-filled pipe[2]. We obtained an analytical result for guided waves that propagate in a cylindrical pipe, whose wave number was a complex number.[3] Pavlakovic theoretically investigated leaky guided waves propagating in a multilayer cylindrical pipe, and he chose the Hankel function for the liquid core[4]. However, Hankel functions are not suitable for the liquid core because they diverge at the center axis of the pipe. In this article, we analyzed the leaky guided waves propagating in a water-filled pipe, and discussed the effect of the attenuation of fluid.

2. Theory of leaky guided wave

Fig. 1 shows the theoretical model of a cylindrical pipe and its coordinate system (cylindrical coordinates). The displacement $\mathbf{u}^{\text{solid}}$ of the pipe ($a \le r \le b$) and the displacement $\mathbf{u}^{\text{fluid}}$ of a fluid ($0 \le r \le a$) are represented by a vector ($\mathbf{H}$) and scalar potential (ϕ_s , ϕ_f) as follows

$$\mathbf{u}^{\text{solid}} = \nabla \phi_s + \nabla \times \mathbf{H} \qquad (1)$$
$$\mathbf{u}^{\text{fluid}} = \nabla \phi_f$$

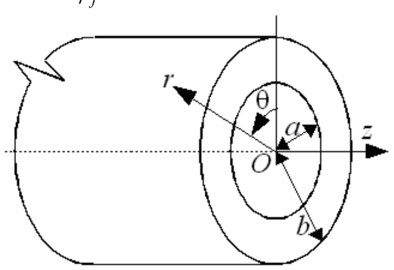

Fig. 1 Theoretical model

The potentials are as follows.

$$\phi_s = f_s(r)\cos n\theta \exp i(k^* z - \omega t)$$
$$H_r = g_r(r)\sin n\theta \exp i(k^* z - \omega t + \pi/2)$$
$$H_\theta = g_\theta(r)\sin n\theta \exp i(k^* z - \omega t + \pi/2)$$
$$H_z = g_3(r)\sin n\theta \exp i(k^* z - \omega t) \qquad (2)$$
$$\phi_f = f_f(r)\cos n\theta \exp i(k_f^* z - \omega t)$$
$$k^* = k(1 + i\eta), \quad k_f^* = k_f(1 + i\eta_f)$$
$$k = \omega/V, \quad k_f = \omega/(V - v)$$

t, k, ω, n, i, η, η_f, V, and v represent time, the wave number of the guided wave propagating in a pipe, the angular frequency, the circumferential mode parameter, the imaginary unit, an attenuation constant of the pipe, an attenuation constant of the fluid, a phase velocity, and a flow velocity of the fluid. By eq. (2) and wave equations of potentials, the Bessel's differential equations are obtained. The solutions to this equations are linear combinations of the Bessel functions (J_n, Y_n), the modified Bessel functions (I_n, K_n), or the Hankel functions ($H_n^{(1)}$, $H_n^{(2)}$). It is decided by the theoretical model, which function is selected as the solution. For example, for leaky guided waves from a water-loaded hollow pipe, the function must become zero at $r \to \infty$ (Fig. 2a). Then, the Hankel function should be selected. For the pipe filled with water (Fig. 2b), the function must not diverge at $r = 0$. Then, the Hankel functions, the Bessel functions of the second kind, and the modified Bessel functions of the second kind cannot be selected for the solution. By this reason, the Bessel functions of the first kind (J_n) or modified Bessel functions of the first kind (I_n) should be selected.

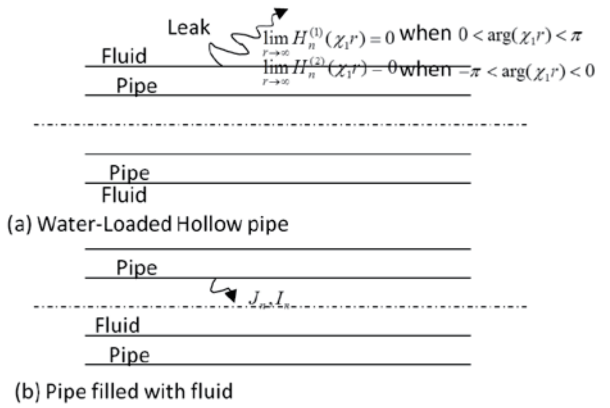

Fig. 2 Selection of the Bessel functions

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The boundary conditions are as follows.

$$u_r^{\text{solid}} = u_r^{\text{fluid}}, \quad \sigma_{rr}^{\text{solid}} = \sigma_{rr}^{\text{fluid}},$$
$$\sigma_{r\theta}^{\text{solid}} = \sigma_{rz}^{\text{solid}} = 0 \quad \text{at} \quad r = a \qquad (3)$$
$$\sigma_{rr}^{\text{solid}} = \sigma_{r\theta}^{\text{solid}} = \sigma_{rz}^{\text{solid}} = 0 \quad \text{at} \quad r = b$$

$\boldsymbol{\sigma}^{\text{solid}}$ and $\boldsymbol{\sigma}^{\text{fluid}}$ are the stress tensors of the pipe and fluid, respectively. By eq. (3) a homogeneous systems of linear equations is obtained.

$$[c_{ij}]\mathbf{x} = 0 \qquad (4)$$

$\mathbf{x}$ is a vector of coefficients of f_s, g_1, g_2, g_3, and f_f. $[c_{ij}]$ is shown in Ref. 3. Dispersion curves are obtained by the following equation.

$$\det[c_{ij}] = 0 \qquad (5)$$

Fig. 3 show absolute values of $\det[c_{ij}]$ at 200kHz. x-axis indicates velocity, and y-axis indicates logarithm of the product of the attenuation ratio and wave number $\log_{10}(\text{Im}(k_f{}^*)) = \log_{10}(\eta_f k_f)$. The parameters used in the calculation are as follows:

$$a = 2\,\text{mm}, \quad b = 3\,\text{mm}, \quad v_l = 5790\,\text{m/s},$$
$$v_t = 3100\,\text{m/s}, \quad \rho_s = 7910\,\text{kg/m}^3$$
$$v_f = 1500\,\text{m/s}, \quad \rho_f = 1000\,\text{kg/m}^3$$

For $n = 0$, Fig. 3a shows that only the solution of $\eta_f = 0$ exists about 1444 m/s, and Fig. 3b shows that the solution of $\eta_f = 0$ and local minimum exist about 4870 m/s and 4850 m/s. For $n = 1$, Fig. 3c shows that the solution of $\eta_f = 0$ and local minimum exist about 1890 m/s and 2060 m/s, respectively. The difference between V of $\eta_f = 0$ and velocity of those local minimums are about 0.3 — 8.8%. The productions of attenuations and wave numbers ($\text{Im}(k_f{}^*) = \eta_f k_f$) of the local minimum areas are about 10^4—10^6 m^{-1}, and the displacements attenuate e^{-1} times par 1—100 μm. Then even though the guided waves whose phase velocities are within the local minimum areas exist, it is difficult to detect them experimentally. On the other hand, we detected the guided waves which propagated 313 mm experimentally (Fig. 4). Then, we consider that the guided waves which detected by the experiment should be guided waves of $\eta_f = 0$.

We discussed about the analysis of the leaky guided wave. The effect of the attenuation of fluid seems negligibly small for this article's situation. However, more numerical results in the different situations are required in order to analyze leaky guided waves.

Acknowledgment
This work was supported by JSPS KAKENHI Grant Number 26390109.

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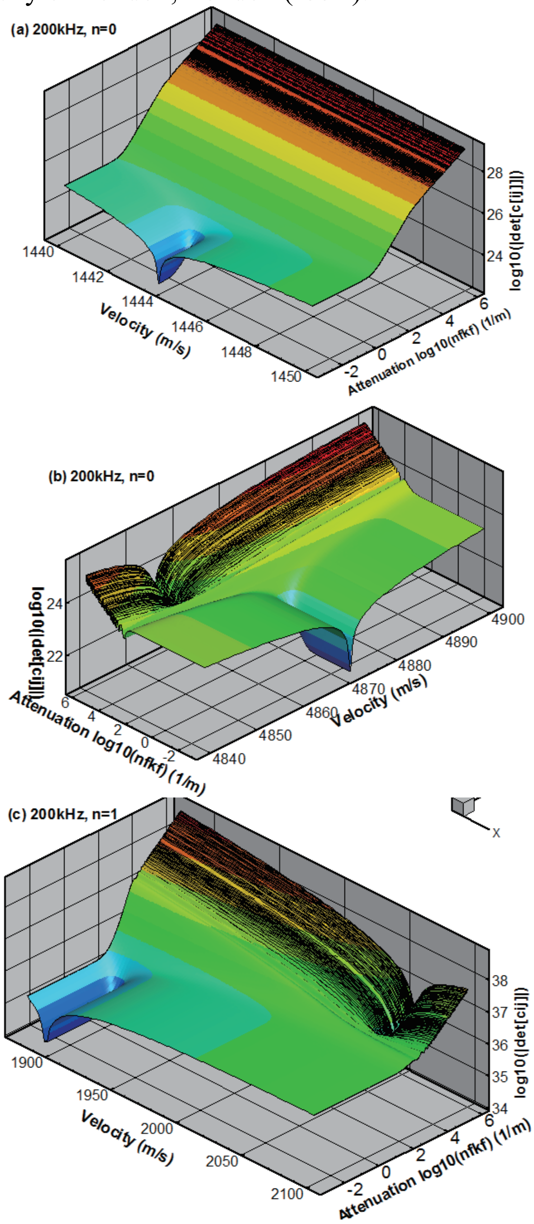

Fig. 3 Absolute value of $\det[c_{ij}]$

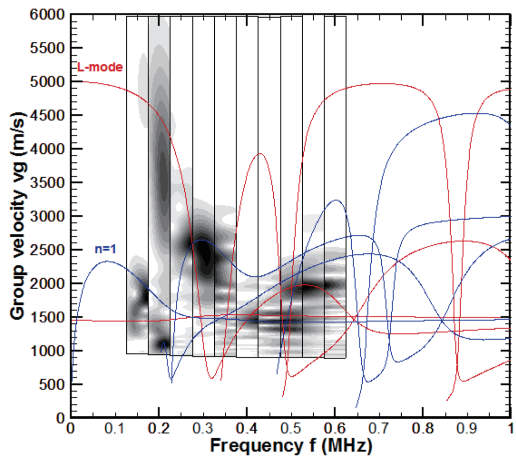

Fig. 4 Group velocities and experimental results