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Simulation of Leaky Lamb Wave Propagation with a Semi-Analytical Finite Element Technique

半解析的有限要素法を用いた漏洩ラム波の
伝搬シミュレーション

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1. Introduction

Ultrasonic wave modes propagating in plate-like structures in the longitudinal direction, called guided waves, can travel long distance with less attenuation than bulk waves. This prominent feature enhances our expectancy for rapid long-range non-destructive testing (NDT). However, in actual inspection, guided waves often attenuate due to energy leakage from the structure to the surrounding media. For the purpose of precise testing, we need to investigate guided wave propagation in a plate with leaky media.

Since guided waves propagate with very complex features such as multi-modal nature and dispersion, numerical calculations are needed to clarify the propagation of guided waves. Although the finite element method (FEM) is often used for calculation of ultrasonic wave propagation, this requires that an object be divided into small elements and is unsuitable for guided waves in large structures due to enormous computation time and memory.

We have developed a numerical calculation method for guided waves using a semi-analytical finite element (SAFE) technique. In SAFE, the wave fields are expressed by orthogonal functions in the longitudinal direction instead of being represented by a number of elements. Thus we can reduce the calculation size significantly.

This paper describes leaky Lamb wave simulations with SAFE. Calculation results are visualized to help our understanding of leaky Lamb waves generated by a point source on a plate.

2. Derivation of transient wave by SAFE

Figure 1 shows the SAFE model used in this study. We consider a two-dimensional plane-strain problem with y and z as spatial coordinates in the thickness and the propagation directions, respectively.

In the fluids, plane harmonic waves are generated by vibration of the plate and radiate to infinity, where the wavenumber ξ_f is given by the ratio of the angular frequency ω and the sound speed of the fluid c_f, as $\xi_f = \omega / c_f$. The plane waves are expressed as $\exp(i\xi_y y + i\xi_z z - i\omega t)$, in

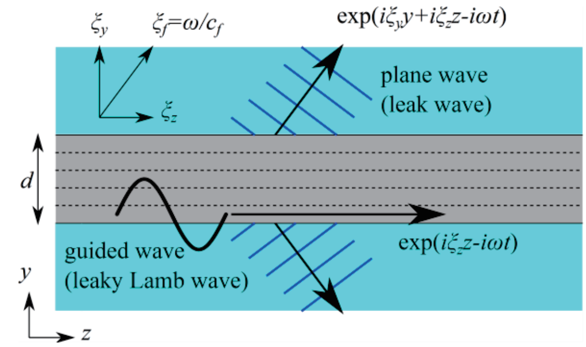


Fig.1 The SAFE model of leaky Lamb wave.

which ξ_y and ξ_z are the y and z components of the wavenumber of the plane waves, respectively. Here, the z component of the wavenumber ξ_z is a common parameter with the wavenumber of Lamb wave. Since we do not discretize the fluid regions in this technique, the calculation size remains the same in the presence of leaky media.

In SAFE, we assume that displacements can be expressed with temporally and spatially harmonic wave $\exp(i\xi_z z - i\omega t)$ in the propagation direction. Since we do not divide an object into small elements in the propagation direction, this technique is suitable for guided waves in large structures. The governing equation, resulting in an eigenvalue problem, provides eigenvalues and eigenvectors that correspond to the wavenumbers ξ_{ym}, ξ_{zm} and the displacement distributions $\mathbf{q}_m$ $(m = 1,\ldots,n)$, respectively, where n is an integer relating to the number of nodes.

An arbitrary nodal displacement vector $\bar{\mathbf{u}}(\omega, \xi_z)$ can be represented by a linear superposition of eigenvectors $\mathbf{q}_m$ [1], as

$$\bar{\mathbf{u}}(\omega, \xi_z) = \sum_{m=1}^{n} \alpha_m \mathbf{q}_m . \tag{1}$$

The coefficients α_m are algebraically determined for different excitation conditions such as point source, piston source and angle beam incidence.

Once we obtain the solution in the wavenumber domain, we carry out the integration using the residue theorem for the kernel $\alpha_m \mathbf{q}_m$ with poles. For example, the displacement field in the space domain for point source incidence at $z = z_0$ is given as,

$$\mathbf{u}(\omega, z) = \sum_{m=1}^{n} \alpha'_m \mathbf{q}_m \exp\{i\xi_{zm}(z - z_0)\}, \tag{2}$$

where the coefficients α'_m are derived from α_m in the process of integration. Moreover, converting from the frequency domain to the time domain, we obtain the solution in time and space domains and visualize the wave propagation by making images of displacement fields at each time step.

3. Simulation result of wave propagation

This section describes one example of the calculation for leaky Lamb wave generated at a point source on a plate surface. A burst wave with the normalized central frequency of $f_c d / c_T = 0.5$ is applied on a plate surface as shown in Fig.2, where f_c is the central frequency, d is the thickness of the plate and c_T is the transverse wave speed in the plate. Displacement field in the region depicted as a red rectangle in Fig.2 is visualized at three different normalized time $f_c t$ in Fig.3. Figure 4 shows the normalized group velocity c_g / c_T versus the normalized frequency $f_c d / c_T$ in addition with normalized frequency spectrum of the excitation waveform. In Fig.3 the grid deformation shows displacements at the grid points and the color denotes the displacement in the propagation direction, in which red and blue stand for positive and negative values of the displacement, respectively. As the wave has very large displacements in Fig.3(c), the displacement field is shown in 1/10 scale of those in Fig.3(a) and (b).

The fastest mode with uniform displacement distribution over the cross-section of a plate shown in Fig.3(a) can be estimated as an S_0 mode from its group velocity and the wave structure. Figure 3(b) clearly shows that plane waves leak out to upper and lower fluids from the S_0 mode. Figure 3(b) also visualizes the A_0 mode as a slower mode following the S_0 mode, which has apparently different wave structure with a node at the plate center.

Another mode appears behind the A_0 mode as shown in Fig.3(b) and (c). The wave propagating along a plate-fluid interface, called Scholte wave[2] has slightly lower wave speed than that of the fluid. The vibration of Scholte wave concentrates in the fluid region close to the plate surface and does not attenuate while propagation. The non-dispersive and non-attenuative characteristics could be suitable for a wide-range inspection of plate-like structures.

4. Conclusion

This paper analyzed transient wave motion in a plate with leaky media numerically with a semi-analytical finite element (SAFE) technique. We first described the formulation of SAFE for leaky Lamb waves generated by a point source.

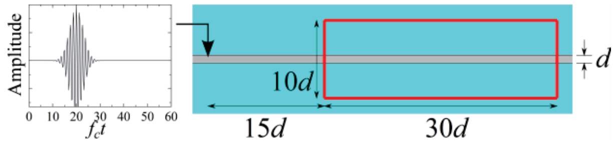
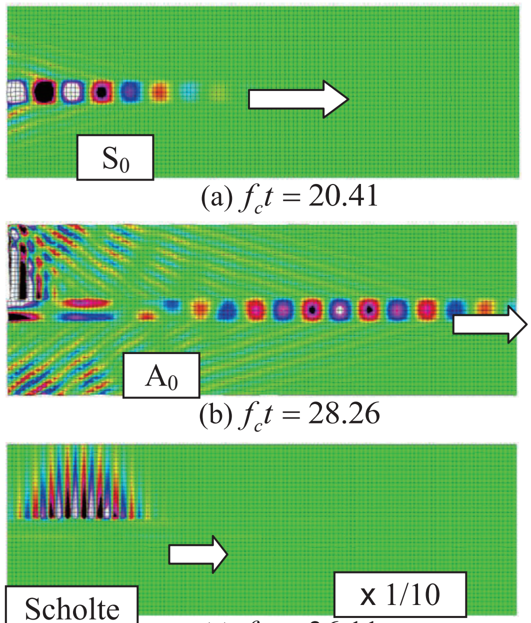


Fig.2 The excitation waveform and visualization region.



(a) $f_c t = 20.41$

(b) $f_c t = 28.26$

(c) $f_c t = 36.11$

Fig.3 Snapshots of displacement fields at three different time steps.

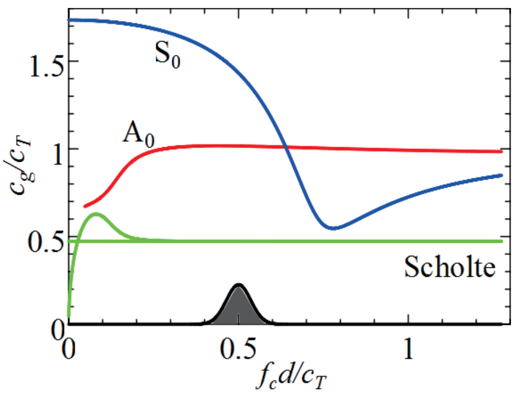


Fig.4 Group velocity dispersion curves and frequency spectrum of excitation waveform.

Calculation results were visualized to analyze fundamental Lamb modes and Scholte wave propagation clearly. As a result, Scholte wave propagates with its displacement mainly in the fluid region close to the plate surface at the speed almost the same as the sound wave in the fluid.

References

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