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Extraction of Particle Size Distribution and Surrounding Ultrasonic Pressure in Water from Laser Diffraction Pattern

レーザ回折パターンからの水中に分散する粒子の粒度分布とその周辺音圧の抽出

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1. Introduction

Ultrasonic pressure in water is general interest in the application of ultrasonics. The ultrasonic pressure is measured by employing a light1,2), however, the light probe has not been applied to the ultrasound field with light scatterers, such as solid particles, microbubbles, and acoustic cavitations. Moreover, it is highly desirable if we can measure the size distribution of optical scatterer, such as acoustic cavitation in sonoreactor3), in addition to the ultrasonic pressure.

In this paper, we perform measurements of the pressure amplitude of the one-dimensional ultrasonic standing wave and the size distribution of the particles dispersed in the ultrasound using laser diffraction. The pressure amplitude is measured from the fluctuation of the laser beam passing the ultrasound4) and the particle size distribution is measured based on the method employed in the laser diffraction particle size analyzer.5)

2. Measurement theory

Figure 1 shows the optical Fourier transform system. When a laser beam diffracted by an ultrasound and a particle, the light amplitude on the focal plane of the Fourier transform lens, $A(x, y)$, is expressed as,

$$A(x,y) = H[x/(\lambda f), y/(\lambda f)] \tag{1}$$

$$\begin{aligned} H(\nu_x,\nu_y) &= F[b(x,y)t(x,y)l(x,y)] \\ &= B(\nu_x,\nu_y) * T(\nu_x,\nu_y) * L(\nu_x,\nu_y), \end{aligned} \tag{2}$$

where λ, f, ν_x, ν_y, $F[]$, $*$, $L(\nu_x, \nu_y)$, $T(\nu_x, \nu_y)$, and $B(\nu_x, \nu_y)$ are the optical wavelength, the focal length of the Fourier transform lens, the spatial frequency of x- and y-direction, the Fourier transformation, the spatial convolution, the Fourier transformation of the incident laser beam, $l(x, y)$, the Fourier transformation of the complex transmittance of the ultrasound, $t(x, y)$, and the particle, $b(x, y)$, respectively.

The one-dimensional ultrasonic standing wave, $p(x, \theta)$, is expressed as

$$p(x,\theta) = p_0 \sin(kx+\phi)\sin(\theta), \tag{4}$$

where p_0, k, ϕ, and θ are the global amplitude of sound pressure, the ultrasonic wavenumber, the spatial initial phase, and the temporal phase, respectively. Equation (4) can be expanded to the power series about the laser beam incident point, x_0, as

$$p(x,\theta) = \sum_{n=0}^{\infty} c_n (x-x_0)^n \sin(\theta). \tag{5}$$

When the laser beam adequately narrower than the ultrasonic wavelength enters parallel to the ultrasonic wavefront, the diffracted light, $T(\nu_x, \nu_y) * L(\nu_x, \nu_y)$, becomes

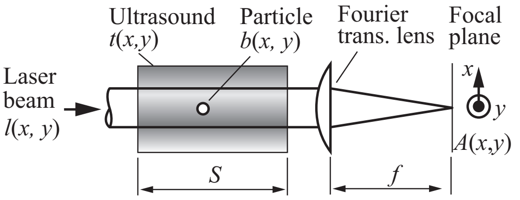

Fig. 1 Optical Fourier transform system.

$$\begin{gathered} T(\nu_x,\nu_y) * L(\nu_x,\nu_y) = \frac{l_0 \pi}{\sqrt{\alpha\beta}} \exp\left[-\frac{\pi^2 (\nu_x - c_1\kappa)^2}{\beta} - \frac{\pi^2 (\nu_y)^2}{\alpha} \right], \\ \alpha = 4/\rho^2, \beta = \alpha - \mathrm{j}\kappa c_2, \kappa = S\gamma/\lambda, \end{gathered} \tag{6}$$

where l_0, ρ, j, S, and γ are the maximum amplitude of the laser beam, the diameter of the laser beam, the imaginary unit, the path length of the laser beam in ultrasound, and the acousto-optic coefficient, respectively.4)

The Fourier transformation of the complex transmittance of a spherical particle, $B(\nu_x, \nu_y)$, is

$$B(\nu_x,\nu_y) = \begin{cases} \delta\left(\sqrt{\nu_x^2+\nu_y^2}\right) - \dfrac{a^2}{2} & \sqrt{\nu_x^2+\nu_y^2} = 0 \\ -4\left(\dfrac{a}{2}\right)^2 \dfrac{J_1\left(2\pi a\sqrt{\nu_x^2+\nu_y^2}\right)}{2\pi a\sqrt{\nu_x^2+\nu_y^2}} & \sqrt{\nu_x^2+\nu_y^2} \neq 0 \end{cases} \tag{7}$$

where a and δ are the diameter of the particle and the Dirac delta function, respectively. The light amplitude on the focal plane becomes convolution between Eqs. (6) and (7). When the laser beam diameter is adequately larger than that of the particle, the light amplitude near the light axis can be approximated by $T(\nu_x, \nu_y) * L(\nu_x, \nu_y)$ and the light amplitude apart from the light axis is similar to $B(\nu_x, \nu_y)$.

When the laser beam crosses to the plural particles, the light intensity on the focal plane, $I(x, y)$, apart from the light axis becomes the superposition of the intensity of the light diffracted by each particle and expressed as

$$I(x,y) = C\sum_{a=0}^{\infty} l_0 v(a) a^4 \left\{ \frac{J_1\left[2\pi a/(\lambda f)\sqrt{x^2+y^2}\right]}{2\pi a/(\lambda f)\sqrt{x^2+y^2}} \right\}^2, \tag{8}$$

where $v(a)$ and C are the volume based particle proportion, and the proportional coefficient, respectively. The volume based particle proportion (size distribution) can be measured by solving inverse problem based on this equation5). The light intensity near the light axis is similar to the square of Eq. (7) as

$$I(x,y) = \frac{(l'\pi)^2}{\alpha\beta} \exp\left\{ -\frac{2\pi^2[x/(\lambda f) - c_1\kappa]^2}{\beta} - \frac{2\pi^2[y/(\lambda f)]^2}{\alpha} \right\}, \tag{9}$$

where l' is the maximum light amplitude of the laser beam decayed according to Beer-Lambert's law for scattering by the particles. By the fluctuation amplitude of c_1 and c_2 from the measured diffracted light, the

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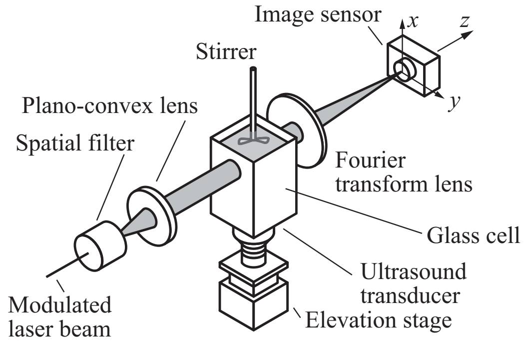
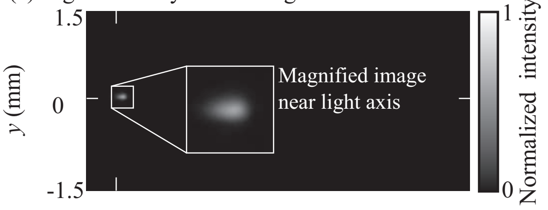
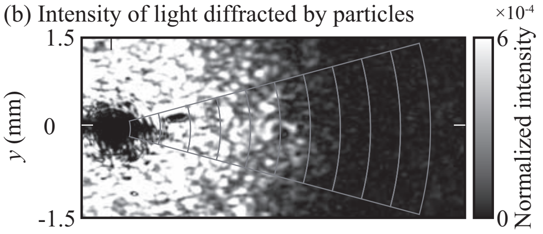


Fig. 2 Experimental setup.

(a) Light intensity near the light axis



(b) Intensity of light diffracted by particles



(c) Intensity averaged in sectorial region shown in (b)

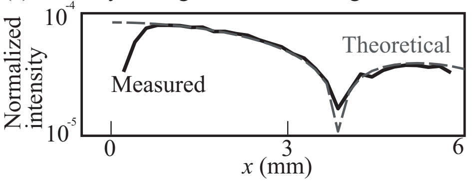


Fig. 3 Diffraction pattern (a) near the light axis (b) apart from light axis.

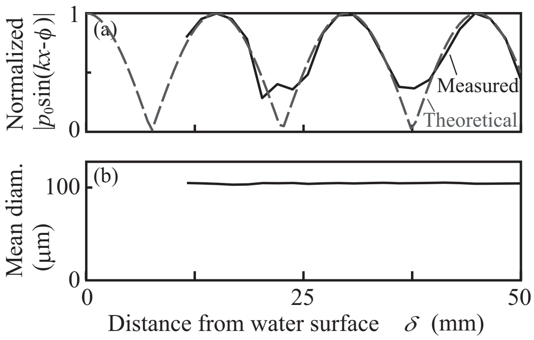


Fig. 4 Spatial distribution of (a) normalized pressure amplitude and (b) mean diameter of particles.

pressure amplitude $p_0\sin(kx+\phi)$ can be obtained.[4)]

3. Experimental setup

Figure 2 is a schema of experimental setup. One-dimensional ultrasound field is established by irradiating ultrasound into water from an ultrasound transducer attached to bottom of a glass cell. The inner dimensions of glass cell is 50×50×160 mm^3. The transducer is driven by a sinusoidal voltage of 48.1 kHz frequency. The water temperature is 25°C, and the ultrasound wavelength is 31.9 mm. The input power to the transducer is 18 W. Well-sorted glass beads of 100 μm in diameter are added to the water and uniformly dispersed by rotating stirrer.

A laser beam with 632.8 nm vacuum wavelength is modulated by an acousto-optic modulator to generate a pulse laser beam with 100 ns duration. The modulated laser beam is spatially filtered and collimated to 3 mm diameter. The collimated beam is introduced into water parallel to the ultrasonic wavefront and outgoing beam enters a Fourier transform lens. The light intensity on the focal plane of the lens is measured by an image sensor.

4. Experimental results

Figures 3(a) and 3(b) show the light intensities on the focal plane measured with and without attenuation of 1/8000, respectively. These were measured at 13.2 mm from the water surface the phase of the transducer driving voltage 0 rad. In Fig. 3(b), the background light intensity is subtracted and the light intensity near the light axis could not be measured because of the saturation of the image sensor. Figure 3(c) shows the intensity averaged in the sectorial region schematically illustrated in Fig. 3(b). This figure also shows the theoretical diffracted light intensity for the particle of 100 μm diameter. The shape of the radial intensity and the position of the dip well agreed.

Figure 4(a) shows the spatial distribution of the pressure amplitude calculated from the fluctuation of the light intensity near the light axis measured with attenuation represented in Fig. 3(a). The theoretical pressure distribution is also plotted. The water surface becomes a pressure anti-node and the pressure anti-node and node are formed alternatively quarter ultrasonic wavelength. The position of the pressure antinode well agreed with theoretical value. The pressure node was not clear but this tendency is agreed with the previously reported result[4)]. Figure 4(b) shows the measured mean diameter of particles calculated from the diffracted light intensity represented in Fig. 3(b). The mean diameter agreed precisely with the intrinsic diameter of the glass particles. Thus, it was shown that the ultrasound pressure of the one-dimensional ultrasound and the size of the particle dispersed in the ultrasound can be measured from the diffracted light intensity.

5. Conclusion

The pressure amplitude of the one-dimensional ultrasound and the size of the particle dispersed in the ultrasound were extracted from the intensity distribution of the diffracted laser beam. The measurement result well agreed with the theoretical value and the previously reported result. The simultaneous measurement of the pressure amplitude and particle size distribution will be achieved by employing the laser diffraction.

Acknowledgment

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References

1) L. E. Hargrove *et al*: J. Acoust. Soc. Am. **31** (1959) 1366.
2) S. Harigane et al.: Jpn. J. Appl. Phys. **52** (2013) 07HF07.
3) T. G. Leighton, Ultrason. Sonochem. **2** (1995) S123.
4) T. Kuroyama *et al*.: Jpn. J. Appl. Phys. **53** (2013) 07KE12.
5) T. Kuroyama *et al*.: Jpn. J. Appl. Phys. **52** (2012) 07HE15.