Transcranial Doppler ultrasonography using spatial domain interferometry with Capon method: simulation study

Beamspace Capon 法を用いた頭蓋内超音波血流速度推定法

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1. Background

After the occurrence of the subarachnoid hemorrhage (SAH), the control of vasospasm is important. Transcranial Doppler ultrasonography (TCD) is a non-invasive test that detects the vasospasm by measuring the blood flow velocity in the middle cerebral artery. However, an interference returned from a cranium deteriorates the accuracy of TCD. We have proposed a TCD method that suppresses the contribution of interferences using the spatial domain interferometry (SDI) with the element space (ES) Capon method [1]. Typically, SDI with ES capon method using many elements causes computational complexity because the size of the spatial covariance matrix increases as the number of elements increases. To reduce the computational load, we employ the beam space (BS) Capon method [2]. The BS Capon method reduces the computational complexity by generating and selecting beams, where a spatial covariance matrix is calculated using selected beams. In this study, we investigate the proposed TCD method with BS Capon method in a numerical simulation.

2. Methods

2.1 SDI with temporal and axial direction averaging

To suppress the interferences, we apply a moving target indicator (MTI) filter and the SDI method with the Capon method [1]. The SDI with the Capon method requires estimation of covariance matrix \mathbf{R} that expresses the cross-correlation between two signals at two elements.

To acquire a high temporal resolution, we averaged **R** along both temporal and axial direction. The (i,j)-th element of **R** is expressed by

$$[\mathbf{R}]_{i,j} = \frac{1}{mT_{\text{ave}}} \sum_{k=1}^{m} \int_{T_{\text{obs}}-T_{\text{ave}}/2}^{T_{\text{obs}}+T_{\text{ave}}/2} x_{k,i}^{*}(t) x_{k,j}(t) \mathrm{d}t , \qquad (1)$$

where $x_{k,l}(t)$ is the *k*-th differential signal of the *l*-th element after time-delay compensation for focusing, *m* is the number of temporal averaging, $cT_{obs}/2$ is the desired range, $cT_{ave}/2$ is the average length in the axial direction, and *c* is the sound velocity.

The Capon method calculates the optimal weighting vector with following optimization problem:

min $P_{out} = \mathbf{W}^{H}\mathbf{R}\mathbf{W}$ subject to $\mathbf{C}^{H}\mathbf{W} = 1$, (3) where **C** is a constraint vector, **W** is a weighting vector, and []^H denotes conjugate transpose. The optimal weighting vector calculated by the ES Capon method is given by

$$\mathbf{W}_{\text{opt}} = \frac{1}{\mathbf{C}^{\mathsf{H}} \mathbf{R}^{-1} \mathbf{C}} \mathbf{R}^{-1} \mathbf{C} \,. \tag{4}$$

Note that the complexity of the matrix inversion is $O(L^3)$. The complexity thus mainly depends on the number of elements used for estimation. The output of SDI with ES Capon method is expressed as follows:

$$y_{\rm ES}(t) = \mathbf{W}_{\rm opt}^{\rm H} \mathbf{x}_k(t)$$
(5)

where $[\mathbf{x}_k(t)]_i$ is $x_{k,i}(t)$.

2.2 SDI with beamspace adaptive beamforming

The BS Capon method decreases the complexity by generating and selecting beams, as shown in **Fig. 1**. The BS Capon method with *L* elements first generates the *L* orthogonal beams by multiplying the Butler matrix to MTI filtered signal. Butler matrix **B** and BS received signal at *k*-th transmit event $\mathbf{x}_{BSk}(t)$ are expressed as follows:

$$\left[\mathbf{B}\right]_{i,j} = \frac{1}{\sqrt{L}} \exp(j2\pi i j / L) , \qquad (6)$$

$$\mathbf{x}_{\mathrm{BSk}}(t) = \mathbf{B}\mathbf{x}_{k}(t) \,. \tag{7}$$

Therefore, Eq. (3) and (4) are rewritten as follows: $\min P_{\text{out}} = \mathbf{W}_{\text{BS}}^{\text{H}} \mathbf{R} \mathbf{W}_{\text{BS}}$ subject to $\mathbf{W}_{\text{BS}}^{\text{H}} \mathbf{C}_{\text{BS}} = 1$ (8)

$$\mathbf{W}_{\rm BS} = \frac{\mathbf{R}_{\rm B}^{-1} \mathbf{C}_{\rm B}}{\mathbf{C}_{\rm B}^{-1} \mathbf{R}_{\rm B}^{-1} \mathbf{C}_{\rm B}},\tag{9}$$

$$\mathbf{R}_{\rm B} = \mathbf{B}\mathbf{R}\mathbf{B}^{\rm H},\tag{10}$$

$$\mathbf{C}_{\mathrm{B}} = \mathbf{B}\mathbf{1},\tag{11}$$

where \mathbf{R}_{B} is the BS covariance matrix, \mathbf{C}_{B} is the BS covariance matrix, and $\mathbf{1}=[1 \ 1 \cdots 1]$. When we select *N* beams for BS Capon method, the complexity of inversion is $O(N^{3})$.

Because the echo from the desired direction is received when the beam direction is close to the desired direction, we select a few



Fig. 1 Schematic of BS Capon

received data sets with the beam directions close to the desired direction. This process reduces the computational load for calculating the optimal weighting vector. The output of SDI with BS Capon method is expressed as follows:

$$y_{\rm BS}(t) = \mathbf{W}_{\rm BS}^{\rm H} \mathbf{x}_{{\rm BS}k}(t) \,. \tag{12}$$

3. Results

3.1 Simulation setting

We evaluated our proposed method in a numerical simulation. We used a linear array probe composed of 64 elements. The center frequency of the pulse was 2.0 MHz, the bandwidth of the pulse was 50%, and the axial and temporal average lengths were 0.75 mm and 0.7 ms, respectively. The interest depth was 50 mm. We supposed that the desired signal arrived from front of the probe and true blood flow velocity was 1.0 m/s. The intensity of time varying components was 40 dB lower than the intensity of whole interferences and a cranium returns high-intensity plane wave.

In the BS Capon method, we used only three received data sets, that is N = 3. The beam directions of the three data sets were 0, 1.8 and -1.8 degrees, where the desired direction is 0 degree.

3. 2 Estimation results

Figure 2 shows the blood flow velocity for 10 samples estimated using conventional DAS beamforming, SDI with ES Capon, and SDI with BS Capon method, where the cranium interference arrived from an incident angle of 5 degrees.

The DAS beamforming failed to estimate the blood flow velocity when the ratio of the desired signal to interferences (SI_CR) was less than -60 dB. Contrary, both SDI methods using ES and BS Capon methods succeeded to estimate the blood flow velocity within the estimation error of less than 0.053 m/s when SI_CR was from -20 dB to -80 dB. For the



Fig. 2 Estimated blood flow velocity errors using a conventional DAS beamforming, SDI with ES Capon method, and SDI with BS Capon method with proposed covariance estimation method, where the signal intensity from blood cells normalized by the intensity of the cranium interference ranges from -20 to -80 dB. The true blood flow velocity is 1.0 m/s. Error bars show the standard deviation.

calculation of an optimal weighting vector, SDI with the ES Capon method and SDI with the BS Capon method required 1.8 ms and 0.14 ms, respectively. The result shows that SDI with the BS Capon method using three beams largely reduces the calculation time and has the same performance as SDI with the ES Capon method using 64 elements.

4. Conclusion

These results indicate the potential of the SDI with BS Capon method to reduce the computational complexity with maintaining almost the same performance as SDI with ES Capon method. We believe that the proposed method is effective for TCD.

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