## Design of Non-Reciprocal Acoustic Waveguides by Indirect Interband Transitions

間接バンド間遷移を用いた非相反音響導波路の設計

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1. Introduction

Reciprocity is one of the most fundamental propoerties in linear passive systems, where the symmetry of wave propagation is preserved in time and space. To realize unidirectional transmission, i.e, diode characteristic, breaking the time-reversal symmetry is required and may be achieved in non-linear or active systems. Recent demonstrations of acoustic diodes are based on different physical mechanisms, such as non-linear frequency conversion [1], acoustic analogue of Zeeman effect [2], and interfacial mode conversion [3]. Although practical levels of one-way acoustic isolation were achieved in those systems, they naturally suffer from several drawbacks including high input energy and limited bandwidth.

Here, numerically demonstrate we non-reciprocal acoustic waveguides by utilizing the indirect interband transitions between two guided modes. The basic idea was insipred by the recent studies on photonic analogue of the interband transitions between electronic states [4-6], and its acoustic/phononic counterpart has been implimented in different physical systems [7-9]. Our approach may achieve wide-band tunable operation with a low-energy consumption, thus promising for the realization of sophisticated acoustic diode applications.

2. Mode Analysis of Acoustic Waveguides

**Fig. 1(a)** shows a schematic of our proposed non-reciprocal waveguide consisting of a water core sandwiched between two parallel steel plates. To simplify the problem, the y-direction was assumed to be infinitely long, and the core thickness, d, was optimized to be 0.8 mm, such that only two of the lowest guided modes, i.e., m = 0and 1 modes could exist in the system at 1.0 MHz. To design the interband transitions, we firstly carried out mode analysis of the guided wave based on finite element method (FEM). **Fig. 1(b)** shows the numerically simulated dispersion relations of the m = 0 and 1 modes in the un-perturbed waveguide. In the calculation, the sound velocity and mass density of {1480 m/s, 1000 kg/m<sup>3</sup>} and



Fig. 1. (a) Schematic of a non-reciprocal acoustic waveguide consisting of a water core sandwiched between two parallel steel plates. Spatio-temporal modulation of the mass density is applied in the half region at the waveguide center to achieve the one-way mode conversion from the m = 0 ( $\omega_0$ ,  $\beta_0$ ) mode to 1 ( $\omega_1$ ,  $\beta_1$ ) mode. (b) Numerically simulated dispersion relations of the m = 0 and 1 modes (the insets) in the un-perturbed waveguide. The phase matching condition is satisfied only for the forward-propagating m = 0 mode to break the reciprocity (the black arrows).

{5290 m/s, 7860 kg/m<sup>3</sup>} were used for water and steel, respectively. Typical two parallel bands involved in the transition were clearly observed, closely following the analytical result based on  $\beta_m = [(\omega/c_w)^2 - (m\pi/d)^2]^{1/2}$ , where  $\omega$  is the angular frequency and  $c_w$  is the sound velocity in water [10]. Such a parallel bands configuration ensures wide-band operation where the phase matching condition are always satisfied for all incident waves near the designed frequency.

# 3. Non-Reciprocal Wave Propagation by Spatio-Temporal Modulation

A non-reciprocal effect was achieved by

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exploiting the one-way mode conversion from the m = 0 ( $\omega_0, \beta_0$ ) mode to 1 ( $\omega_1, \beta_1$ ) mode, as shown in **Fig. 1(b)**. Such a mode conversion with  $\Delta \omega \neq 0$  and  $\Delta \beta \neq 0$  is referred to as an indirect interband acoustic transition, in analogy with the indirect transitions in semiconductors. To induce the dynamic process, spatio-temporal modulation of the mass density,  $\rho'(t, z) = \rho_w + \Delta \rho \cos(\Delta \omega t - \Delta \beta z)$ , is applied at the waveguide center, where  $\rho_w$  is the water mass density,  $\Delta \rho$  is the modulation depth,  $\Delta \omega$ =  $\omega_1$  -  $\omega_0$  is the modulation frequency, and  $\Delta\beta = \beta_1$ -  $\beta_0$  is the wave-vector shift. From the parity of the m = 0 and 1 mode profiles [the insets of Fig. 1(b)], only the half region within the core  $(-d/2 \le x \le 0)$ was modulated to maximize the efficiency of the mode conversion. In the case of the forward-propagating m = 0 ( $\omega_0, \beta_0$ ) mode, the phase matching condition is satisfied to induce the complete transition to the m = 1 ( $\omega_1, \beta_1$ ) mode. For the backward-propagating m = 0 ( $\omega_0, -\beta_0$ ) mode, on the other hand, the incident wave is maintained due to the phase mismatching, thus realizing the non-reciprocal effect via a linear dynamic process.

**Fig. 2** shows time evolution of the acoustic wave propagation in the non-reciprocal waveguide for the forward- and backward-propagating m = 0 incident wave. In the calculation,  $\Delta \omega = 0.1$  MHz,  $\Delta \beta = 1844 \text{ 1/m}$ ,  $\Delta \rho = 200 \text{ kg/m}^3$ , and L = 16.65 mm were used for the transition from the m = 0 ( $\omega_0/2\pi = 1.0$  MHz,  $\beta_0 = 4308$  1/m) mode to 1 ( $\omega_1/2\pi = 1.1$  MHz,  $\beta_1 = 2464$  1/m) mode. In the case of the forward-propagating wave [Fig. 2(a)-(d)], nearly complete mode conversion was observed. For the backward-propagating wave [Fig. 2(e)-(h)], on the other hand, the m = 0 incident wave was not disturbed at all, passing through the waveguides with a nearly 100% transmittance.

#### 4. Conclusion

A non-reciprocal acoustic waveguide was numerically demonstrated by utilizing the indirect interband transitions between two guided modes. With the spatio-temporal modulation of the guided wave to break the reciprocity, the forward-(backward-) propagating m = 0 mode was (was not) converted to the m = 1 mode, depending on the phase matching condition. Our approach may achieve a high-efficiency acoustic diode with wide-band tunability, thus opening up new avenues for the sophisticated acoustic wave manipulation.

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Fig. 2. Time evolution of the acoustic wave propagation in the non-reciprocal waveguide for the forward- and backward-propagating m = 0 incident wave. The time steps are (a, e)  $t = 3.2 \,\mu$ s, (b, f)  $t = 8 \,\mu$ s, (c, g)  $t = 16 \,\mu$ s, and (d, h)  $t = 40 \,\mu$ s. The one-way mode conversion is clearly observed only for the forward-propagating incident wave. All figures were vertically enlarged 10 times for clarity.

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