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Nulling crosstalk with a time-reversal mirror using the Gram-Schmidt process

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1. Introduction

Time-reversal processing (TRP) has been demonstrated in various fields such as optics, ultrasonic, and underwater acoustics.[1,2] In TRP, a transmitted probe signal is received at array of each source-receive elements, often referred to as a time-reversal mirror (TRM), and the received signals are time-reversed to be backpropagated. If the propagation medium is static, TRP results in a coherent acoustic focusing at the probe source (PS) location.

In adaptive signal processing, minimum varaince distrontionless respnse (MVDR) is well known for minimization of the output power of variance subject to a constraint on the look direction. This adaptive method has been applied to TRP which is called adaptive time-reversal mirror (ATRM) in an ocean waveguide.[3,4]

In this study, the Gram-Schmidt process which is a method for orthogonalizing a set of vectors in an inner product space is applied to TRP to place nulls at the interferer locations while maintaining a distortionless response at the probe source location.

2. Theoretical interpretation and analysis

In ATRM, a signal vector $\mathbf{w}$ for backpropagation can be written as

$$P(\vec{r}) = \sum_i w_i^* \mathrm{G}(\vec{r}|\vec{r}_i) = \mathbf{w}^+ \mathbf{G}(\vec{r}|\mathbf{r}_{\mathbf{array}}) \qquad (1)$$

where superscripts $(\,)^*$ and $(\,)^+$ denote complex-conjugate and Hermitian transpose, respectively. The optimization problem of the objective function in Eq.(2) is used to calculate the weight vector $\mathbf{w}$.

$$\min_{\mathbf{w}} \mathbf{w}^+ \mathbf{K} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^+ \mathbf{G}(\vec{r}_{ps}|\mathbf{r}_{\mathbf{array}}) = 1 \qquad (2)$$

The following constrained properties are satisfied by backpropagating the weight vector $\mathbf{w}$ in case of three probe sources.

$$\mathbf{w}^+ \mathbf{G}(\vec{r}_{n1}|\mathbf{r}_{\mathbf{array}}) = <\mathbf{G}_{n1}, \mathbf{w}> = 0$$
$$\mathbf{w}^+ \mathbf{G}(\vec{r}_{n2}|\mathbf{r}_{\mathbf{array}}) = <\mathbf{G}_{n2}, \mathbf{w}> = 0$$
$$\mathbf{w}^+ \mathbf{G}(\vec{r}_{ps}|\mathbf{r}_{\mathbf{array}}) = <\mathbf{G}_{ps}, \mathbf{w}> = 1 \qquad (3)$$

In Eq.(3), both $\vec{r}_{n1}$ and $\vec{r}_{n2}$ are the locations where the nulls are placed while the location with distortionless response is denoted as $\vec{r}_{ps}$, and $< >$ denotes an inner product in a complex vector space.

Fig.1(a) conceptually depicts the geometry of the Eq.(3), and intuitively shows that the weight vector $\mathbf{w}$ is orthogonal with respect to the acoustic field $\mathbf{G}_{n1}$ and $\mathbf{G}_{n2}$.

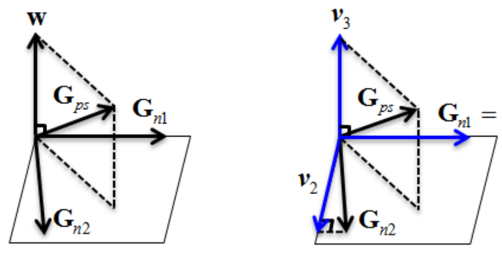

Fig. 1 Geometric visualization of the constrained properties.

Instead of finding the optimal solution of objective function in Eq.(2), the Gram-Schmidt process can construct orthogonal basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ in **Fig.1(b)** from the signal vectors $\{\mathbf{G}_{n1}, \mathbf{G}_{n2}, \mathbf{G}_{ps}\}$ which are assumed as an arbitrary basis for an inner product space. The Gram-Schmidt process then works as

$$\boldsymbol{v}_1 = \mathbf{G}_{n1}$$
$$\boldsymbol{v}_2 = \mathbf{G}_{n2} - \frac{<\mathbf{G}_{n2}, \boldsymbol{v}_1>}{\|\boldsymbol{v}_1\|^2} \boldsymbol{v}_1$$
$$\boldsymbol{v}_3 = \mathbf{G}_{ps} - \left[\frac{<\mathbf{G}_{ps}, \boldsymbol{v}_1>}{\|\boldsymbol{v}_1\|^2} \boldsymbol{v}_1 + \frac{<\mathbf{G}_{ps}, \boldsymbol{v}_2>}{\|\boldsymbol{v}_2\|^2} \boldsymbol{v}_2\right] \qquad (4)$$

The properties of the signal vector $\boldsymbol{v}_3$ give the distortionless response in the look location when backpropagated, which is intrinsically equivalent to the ATRM solution.

3. Results and discussion

Simulations and SWellEx-96 experimental data are combined to demonstrate the placement of nulls at the locations $\vec{r}_{n1}$ and $\vec{r}_{n2}$ with focusing at the location $\vec{r}_{ps}$ by the Gram-Schmidt process in TRP.

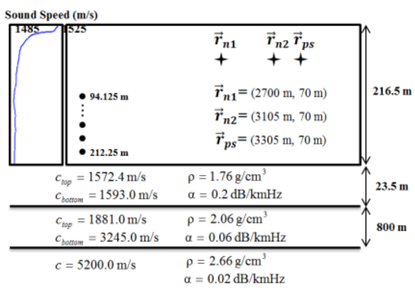

Fig. 2 Schematic of the SWellEx-96 experiment.

We simulate backpropagation using the real data received on the Source-Receive Array (SRA) for environment shown in **Fig.2**. The backpropagated field response for the center frequency 130 Hz is shown in **Fig.3**. The nulls at the locations $\vec{r}_{n1}$ and $\vec{r}_{n2}$ are noted in **Fig.3(a)** and **(b)** as a result of the nulling process when comparing with conventional TRP shown in **Fig.3(c)** and **(d)**.

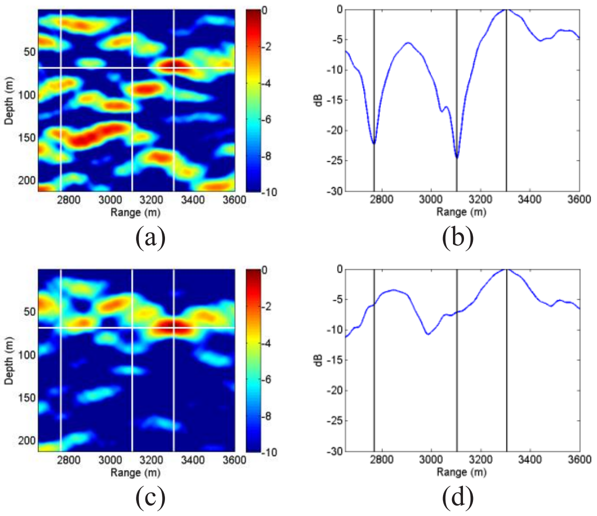

Fig. 3 Simulated single frequency focusing at 130 Hz.

In order to obtain the backpropagated time series, the probe signal with a 0.2 sec pulse at a 130 Hz carrier is used for the numerical simulation. **Figure 4(a)** and **(b)** show the depth-stacked time series at ranges of $\vec{r}_{n2}$ using conventional TRP and the Gram-Schmidt process, respectively, with focusing at ranges of $\vec{r}_{ps}$ as shown in **Fig.4(c)** and **(d)**. It is shown that the null is placed at a depth of 70 m in **Fig.4(b)** since the backpropagated signal vector is guaranteed to be orthogonal with respect to the acoustic field at $\vec{r}_{n2}$.

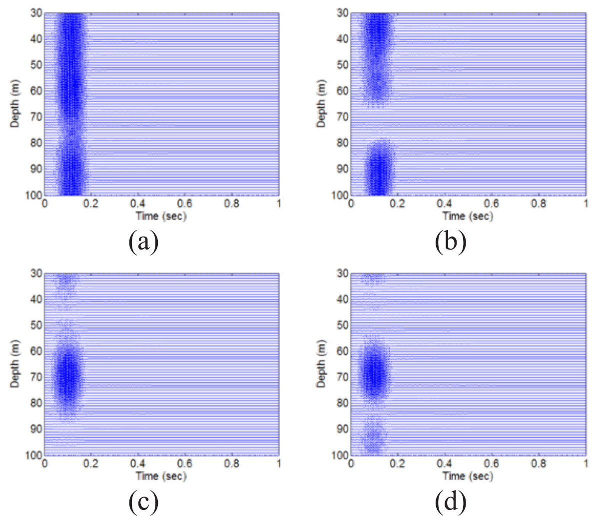

Fig. 4 Simulated backpropagated time series.

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4. Possible application for the method

The method presented can be applied to the multiuser communications in shallow water to effectively remove the crosstalk between users. **Figure 5** shows the performance of communications using FAF-05 experimental data. A detailed description of the experiment can be found in Ref.5.

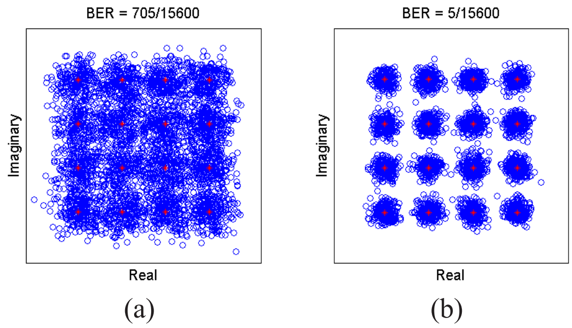

Fig. 5 The scatter plots using 16-QAM.

The result of the conventional TRP is shown in **Fig.5(a)** when comparing with proposed method as shown in **Fig.5(b)**, exhibiting almost error-free performance.

5. Conclusion

We have investigated the placement of nulls using the Gram-Schmidt process in TRP with both experimental data and numerical simulations. It is worth noting that the proposed approach is intrinsically equivalent to adaptive time-reversal mirror because both of methods are based on orthogonality. Also, possible application to multiuser communications for suppression of the crosstalk is discussed.

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