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Finite element analysis of acoustic streaming in a Kundt tube with bended wall

縞状壁面をもつクント管内音響流の有限要素解析

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1. Main text

Acoustic streaming around small particle obstacles, such as cork powder, scattered in an acoustic tube is known to produce peculiar series of vortexes[1]. Particularly, the stripe pattern interval length due to this vortex is not similar to wavelength or boundary layer thickness in air, and is known as Kundt tube problem. Considering industrial use of acoustic nonlinear static force or streaming, understanding of the mechanism of vorteces is quite beneficial.

Many experiment on the particle moves in Kundt tube have been historycally discussed so far[2-3]. The stripe cycle is several milimeter in Hz-kHz frequecy and is dependent on acoustic amplitude and frequency. Adachi et al.[4] performed this experiment within for 135 dB amplitude and 1028 Hz standing wave in 20 mm diameter tube. The stripe cycle and height are 2-3, and 4-5 mm, respectively.

This study discusses the acoustic streaming near artificially settled sinusoidal boundary as an approach to reveal the phenomenon. The acoustic finite element analysis (FEA) considering viscosity is performed to derive the imaginably second order driving force[5] from Reynolds stress. Then, the streaming is calculated through the static fluid FEA with the driving force input. The analyses are performed for different cycle and height of the artificial stripe pattern, and the pattern where the acoustic streaming vorteces are the strongest is compared with the literature value.

2. Governing Equations

Linear acoustic equations considering viscosity are

$$\dot{\mathbf{u}} = -\left(1/\rho\right)\nabla p + \nu\nabla^2\mathbf{u}, \qquad \dot{p} = -\rho c^2 \nabla\cdot\mathbf{u}. \tag{1}$$

Directly solving this equation by solid FEA often encounters numerical instability. When solving in two-dimensional space, converting them into two (P- and S-) wave equations by Helmholtz decomposition is quite effective as

$$\mathbf{u} = -\nabla\varphi + \nabla\times\left(\psi\mathbf{e}_z\right), \tag{2}$$

$$\nabla^2\varphi + k_p^2\varphi = 0,\ \nabla^2\psi + k_s^2\psi = 0. \tag{3}$$

Wave numbers are

$$k_p = \omega\Big/\sqrt{c^2 + j\omega\nu}\,,\ k_s = \sqrt{\omega\big/\left(j\nu\right)}, \tag{4}$$

respectively. Note the stick boundary is considered

$$\int\left(\nabla\varphi - T\nabla\psi\right)d\mathbf{S} = 0, \qquad \int\left(T\nabla\varphi + \nabla\psi\right)d\mathbf{S} = 0,\ T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{5}$$

The driving force, processed from acoustic result,

$$\mathbf{F} = \nabla\cdot \mathrm{Re}\left[\mathbf{u}^t\mathbf{u}^*\right]\Big/2, \tag{6}$$

is input to static Stokes fluid equation as

$$-\nu\nabla^2\mathbf{U} = -(1/\rho)\nabla P + \mathbf{F},\ \nabla\cdot\mathbf{U} = 0 \tag{7}$$

The solution ***U*** is the acoustic streaming, the target of this study.

3. Finite Element model

Fig. 1 shows the FE model of the calculation. The drive frequency (1028 Hz) and amplitude (135 dB) is adjusted to follow the values in the literature [4].

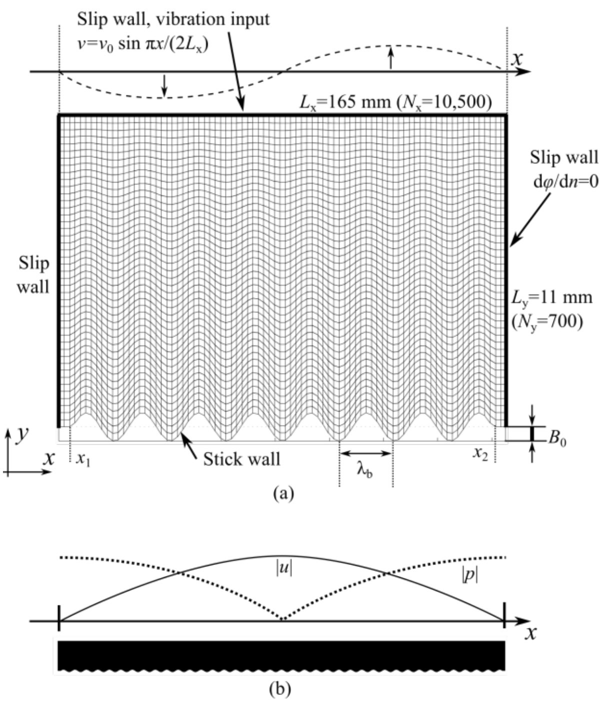

Fig. 1 Finite element (a) concept model and (b) actual calculation mesh with the expecting mode.

The half-wavelength mode air acoustic tube with both closed slip wall is assumed. The acoustic input is upper side slip wall with unsymmetrical normal vibration input. The artificially bended stick boundary is placed bottom side of the tube. Sound speed, density, and kinetic viscosity of air are 340 m/s and 1.29 kg/m3, and 15.1 cSt, respectively. The boundary layer thickness $\lambda_s = 2\pi/\mathrm{Re}[k_s]$= 0.43 mm.

4. Results

Fig. 2 shows x-component acoustic streaming distribution when (a) plane boundary and (b) bended boundary with amplitude of 0.43 mm (λ_s) and stripe cycle of 4.3 mm (10λ_s). Typical Rayleigh type streaming can be seen in both figure, whereas in the streaming (b), many local vortices are generated near each boundary. Fig. 3 shows y-component acoustic streaming near bended boundary at the center of the calculation domain. The vortexes in the size order of λ_s are excited, and are similar to Schlichting streaming by Holtsmark[6].

Fig. 4 shows Maximum y-component acoustic streaming dependency on the stripe cycle and height. In lower height, the streaming characteristic has a peak near cycle λ_s, whereas in the higher boundary of 4 mm (10λ_s) case, the peak is shifted to longer cycle of 2mm (5 λ_s), which agrees with the value in the literature [4].

4. Conclusion

Finite element analyses of Kundt tube with artificial bended boundary are performed for different stripe cycles and heights. Strong local streaming appears near the bended curves when the cycle and height are 2 and 4 mm, respectively, which agrees with the stripe pattern size in the literature.

References

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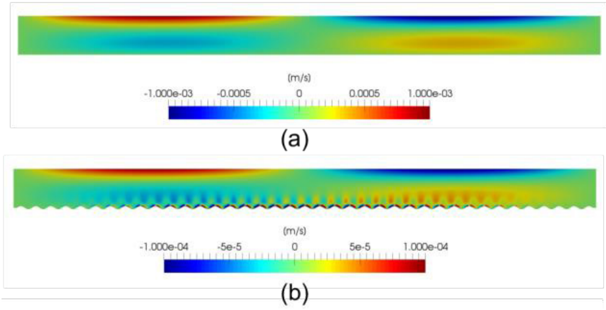

Fig. 2 X-acoustic streaming distribution for (a) plane and (b) bended boundary.

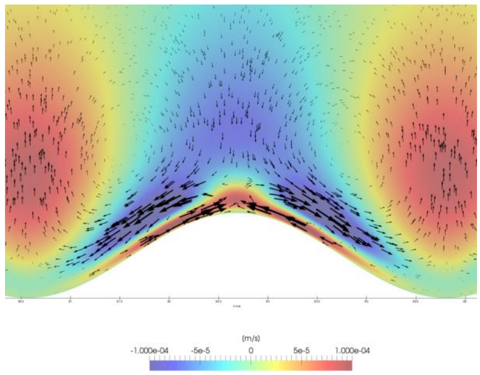

Fig. 3 Y-component acoustic streaming distribution near the central artificial bending.

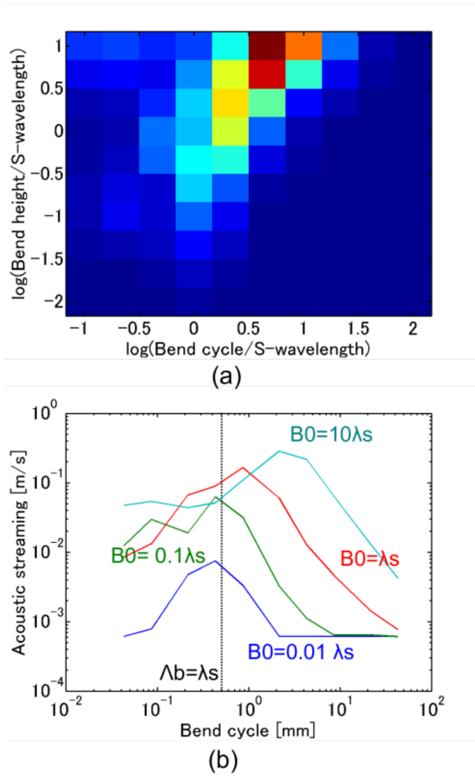

Fig.4 Maximum y-component acoustic streaming dependency on the stripe cycle and height. Note that all the axes are shown in logarithm scale.