# Investigation on Application of Rectangular－Annular Element in Reflection Point Search by Single Sound Source <br> 単一音源による反射点探索における矩形環状要素の適用に関する検討 

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## 1．Introduction

Rectangular transducers are widely used in measuring devices or imaging equipment by ultra－ sonic waves，as elements of the sound source．Rec－ tangular transducers have four vertices and four sides．Depending on the order in which edge waves from these vertices and sides and direct waves from the sound source surface arrive at the observation point，a spatial impulse response of a rectangular sound source complicatedly changes in proportion to the position of the observation point ${ }^{1)}$ ．And，the waveform acquired by a rectangular sound source changes depending on the position of the observa－ tion point，subject to the spatial impulse response． An application of this complicated change to the reflection point search is proposed using a single rectangular sound source ${ }^{2)}$ or a rectangular array sound source with a small number of elements ${ }^{3-8}$ ．

In the conventional methods using these sound sources，the reflection waveform is strongly influenced by the direct wave，when the reflection point is around the perpendicular from the center of the sound source to the observation space，at the position where the direct wave from the surface of the sound source reaches．This is considered to be one of the reasons why good search results of re－ flection points cannot be obtained in that region．

In this study，in order to reduce the influence of the direct wave，a sound source with a rectangu－ lar－annular element is introduced．Improvement of the results of the reflection point search is investi－ gated using numerical calculation of cross－correlation coefficient．

## 2．Method of Reflection Point Search

The configuration of a sound source with a rectangular－annular element and a reflection point $P$ is shown in Fig．1．The sound source is assigned to a plane that is perpendicular to the $z$－axis so that the center of the whole sound source is the origin of the coordinates．The dimension of the whole sound source is $2 a_{o} \times 2 b_{o}$ ．The dimension of the aperture in the central part of the sound source is $2 a_{i} \times 2 b_{i}$ ， and the width of the rectangular－annular element is $a_{o}-a_{i}$ in the horizontal direction and $b_{o}-b_{i}$ in the


Fig． 1 Configuration of a sound source with a rectan－ gular－annular element and a reflection point $P$ ．
vertical direction．In the numerical calculations de－ scribed later，the values of $a_{i}$ and $b_{i}$ are given as ratios to $a_{o}$ and $b_{o}$ ．The position of the reflection point is indicated by $P(\boldsymbol{r})$ ．In the calculation result showing in the following section，$r$ is expressed using the distance from the center of the sound source $(|\boldsymbol{r}|)$ ，the azimuth angle，and the elevation angle．

When the sound source is driven with uni－ form velocity $v(t)$ ，and when the wave radiated from the sound source is reflected at $P$ ，the output $e(r, t)$ in terms of the reflected wave received at the sound source is expressed as ${ }^{9)}$

$$
\begin{equation*}
e(\boldsymbol{r}, t)=-\frac{k \rho A}{2 c} v(t) * \frac{\partial}{\partial t} h(\boldsymbol{r}, t) * \frac{\partial}{\partial t} h(\boldsymbol{r}, t), \tag{1}
\end{equation*}
$$

where $k$ is the proportionality constant，$\rho$ is the den－ sity of the propagation medium of the sound wave， $A$ is the area of the region in which the reflection point contributes to the reflection，$c$ is the velocity of sound，$h(\cdot)$ is the spatial impulse response of the sound source，and $*$ denotes the convolution inte－ gral．

The rise time of the reflected wave is meas－ urable．Therefore，the value of $|\boldsymbol{r}|$ can be determined in the range expressed as

$$
\begin{equation*}
\frac{c T}{2} \leq|\boldsymbol{r}| \leq \frac{c T}{2}+\sqrt{a_{o}^{2}+b_{o}^{2}} \tag{2}
\end{equation*}
$$



Fig. 2 Calculation results of cross-correlation coefficients at three reflection points: (I) using rectangular-annular sound source with ratio of width of $50 \%$; (II) using single rectangular sound source.
where $T$ is the rise time of the reflected wave, and $c$ is velocity of sound. When the value of $\boldsymbol{r}$ is set at an appropriate interval in the range of $|\boldsymbol{r}|$, the spatial impulse response $h(\boldsymbol{r}, t)$ corresponding to each $\boldsymbol{r}$ can be obtained. Since $v(t)$ is known, the output waveform $e(\boldsymbol{r}, t)$ in eq. (1) at each $\boldsymbol{r}$ can be calculated. By deducing the cross-correlation coefficient between the waveform obtained by the calculation and the original (acquired) reflected wave in the sequential order, it becomes possible to estimate the position of the reflection point $P$.

## 3. Numerical Calculations

The results of numerical calculations by the rectangular-annular sound source are shown in Fig. 2(I). The results are obtained by calculating convolution integral in eq. (1) and the cross-correlation coefficient at time zero with the calculation result for the points around the reflection points sequentially. The dimensions of the sound source used in the calculation are $a_{o}=6.450 \mathrm{~mm}$, and $b_{o}=10.050$ $\mathrm{mm}, a_{i}=3.225 \mathrm{~mm}$, and $b_{i}=5.250 \mathrm{~mm}$. Therefore, the ratio of $a_{i}$ and $b_{i}$ to $a_{o}$ and $b_{o}$ is $50 \%$. For the comparison, the calculation results by a single rectangular sound source which has the dimension of $2 a_{o} \times 2 b_{o}^{2)}$ are also shown in Fig. 2(II).

In Fig. 2(I), fluctuations of the correlation coefficient that appear observably in Fig. 2(II) are generally suppressed. Especially in Fig. 2(I)(b), a conspicuous high correlation at the position set as
the reflection point can be seen. It is considered that the proposed method has some efficacy on the improvement of the search results under certain conditions.

## 4. Summary

In the reflection point search by rectangular sound sources, the sound source with a rectangu-lar-annular element was introduced. The proposed sound source had an aperture in the center part of the sound source element. Calculation results of the correlation coefficient showed that the influence of the direct wave on the search result by the conventional method was suppressed.

In order to obtain a good search result, it is important to appropriate setting of the width of the element, and it is necessary to further investigate.

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