# Theoretical modeling and experimental measurement for bandgap control of phononic crystals

フォノニック結晶のバンドギャップ制御に関する理論モデルと測定

Takahiro Nishino, Atsushi Ishikawa, Kazuhiro Fujimori and Kenji Tsuruta (Okayama Univ.) 西野貴大,石川篤,藤森和博, 鶴田健二 (岡山大院 自然)

## 1. Introduction

Elastic wave in bulk or in surface has been used as transmitting media in many technological applications. Particularly in communication equipment, it is applied to a frequency filter for noise reduction [1]. However, application of the technology to higher frequency domains has been limited, and a new principle is desired especially to achieve precise control of such wave propagations. For this reason, we focus on Phononic Crystals (PnC) [2]. They are periodic structures capable of selectively transmitting and controlling the propagation of elastic waves. They also have bandgap in their vibrational property, where the propagation of elastic waves within a range of frequencies is forbidden in all directions [3]. Since the range of bandgap is scalable to the size of the structure, one can apply the analysis at lower frequencies to designing the device operating at higher frequencies.

In this study, we aimed to establish quantitative analysis and design principle of dispersion relations in PnC with periodically arranged cross holes. Using the Finite Element Method (FEM), we show that the bandgap can be precisely tuned by changing structural parameters in the PnC.

### 2. Analysis model and method

Fig.1 shows the unit cell of cross hole structure consisting of aluminum nitride with structural parameters, *a*=500µm, *b*=450µm,  $c=175\mu$ m,  $h=25\mu$ m, chosen for MHz operations. Aluminum nitride has excellent temperature characteristics and has been widely used for the frequency filters. Many studies have reported on 2D PnCs consisting of columnar material and periodic array of circular holes [4,5]. The geometrical parameters in these structures are only radius and period, whereas cross hole structures extend the degree-of-freedom in the design. In addition, it has been reported that the cross-hole arrays can generate the wider bandgap than that with round holes [6]. Therefore, our approach is to control the bandgap by

changing parameters (a, b, c) and an oblique angle of the cross holes.



**Fig.1** The unit cell of cross hole structure and the geometrical parameters

A set of numerical simulations for the crosshole PnC model was performed by using a commercially available 3D FEM software, COMSOL Multiphysics<sup>®</sup> [7] developed by COMSOL AB. Since the PnC consists of a periodic array of a structure, the Bloch boundary conditions on the boundaries of a single unit cell can be applied directly to the analyses.

## 3. FEM analysis

**Fig.2** shows band structures calculated by the FEM and vibration mode at point A in the band diagram. A large complete bandgap appears in the frequency range of  $4.2 \sim 8.2$ MHz. We confirmed that those vibration modes can be separately identified as two vibration modes, *i.e.* the SH wave displaced in *xy* direction and the Lamb wave displaced in *z* 



**Fig.2** Band structures calculated by FEM (left) and vibration mode at point A (right).

direction.

**Fig.3** shows a relation between the geometrical parameters and the bandgap. Fig.3(a) indicates that the bandgap becomes smaller as angle becomes larger and absent for most of the oblique angle ( $\theta$ ) larger than 20°. In Fig.3(b), maximum bandgap is found around c/a=0.35. Thus it is confirmed that the position and the range of the bandgap can be controlled by structural parameters of the cross holes.



**Fig.3** The relationship between the geometrical parameters and the bandgap: (a)the dependence on oblique angle ( $\theta$ ) and (b) on c/a.



**Fig.4** Comparison between the FEM values of bandgap and the values obtained in the spring-mass model

### 4. Spring-mass model

The vibrational mode at point A (Fig.2) can be replaced by a mechanical model consisting of spring and mass, and the local resonance frequency is calculated from its equations of motion [8]. **Fig.4** shows comparison between the FEM values of the bandgap and resonance frequencies with the *spring*- *mass model.* The purpose of this comparison is to demonstrate possible prediction for position of the bandgap. The results show that the lower edge of bandgap agrees well with the resonant frequencies with this spring-mass model.



**Fig.5** Schematic of setup used for measurement of transmission properties in the PnC devices.

#### 5. Measurement

**Fig.5** shows a schematic of the setup used for testing the PnC devices. The experimental setup consists of a function generator, digital oscilloscope and piezoelectric transducers. Piezoelectric transducers are mounted on either side of the devices to send and receive elastic waves on the sample. In the presentation, we will report details in the results of the experiment.

### 6. Conclusion

The dependence of geometrical parameters in cross-hole PnCs on the bandgaps was examined via numerical simulations. We showed that the bandgap can be tuned precisely by varying the geometrical parameters. From the observations that vibration mode at the lower edge of bandgap was caused by local resonances of the SH mode, we showed that a mechanical model consisting of spring and mass can well reproduce the dependence of the parameters on the band edge calculated by the FEM. It is thus shown that the position and range of the bandgap in cross-hole PnC can be predicted analytically by the spring-mass model.

### References

- 1. Ian C. Hunter *et al.*, IEEE Trans. on Microwave Theory and Tech. **50**, 794-805 (2002).
- 2. M. S. Kushwaha *et al.*, Phys. Rev. B **49**, 2313 (1994)
- 3. J. O. Vasseur *et al.*, Phys. Rev. B **77**, 085415 (2008).
- Y. Achaoui *et al.*, Phys. Rev. B **83**, 104201 (2011).
- 5. R. Pourabolghasem *et al.*, J. Appl. Phys. **116**, 013514 (2014).
- 6. Y. F. Wang et al., J. Appl. Phys. 110, 11 (2011)
- 7. http://www.comsol.com
- 8. J. H. Oh et al., Sci. Rep. 6, 23630(2016).