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Sub-grid Technique for Numerical Simulation of Sound Wave Propagation Combining Constrained Interpolation Profile Schemes

音波伝搬解析のための複数の CIP スキームを用いた
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1. Introduction

To date, numerical analysis for sound wave propagation in time domain has been investigated widely as a result of computer development. Now, the development of accurate numerical schemes is an important technical issue[1].

The constrained interpolation profile (CIP) method, a novel low-dispersive numerical scheme is a type of method of characteristics (MOC) [2-4].

However, new grid systems are required for CIP simulations of complicated heterogeneous media or large-scale simulations of wave propagation. To overcome this problem, sub-grid techniques[5] are proposed for other simulation methods of wave propagation. In the previous study, we have introduced this technique for the type-C and type-M CIP methods, and evaluated the setting of the boundary interface between the course grid and sub grid[6].

The type-M CIP method is a simple technique with smaller memory use and less calculation time required than the type-C CIP method in exchange for accuracy. Therefore, from the point of reduction in the calculation cost, a sub-grid technique for the type-M CIP method is also important.

Subgrids are defined as those smaller than the surrounding grids: we can use suitable multisize grids in an analysis domain according to a sub-grid technique for the CIP-MOC simulation of sound wave propagation.

In this study, we improved on the sub-grid techniques[6] for CIP analysis using generalized CIP (GCIP) schemes[7] and reported the comparison of accuracy and calculation cost.

2. Sub-grid techniques in CIP method

In CIP analysis, the governing equations for linear acoustic fields (a lossless medium) are transformed into advection forms. For example, for the calculation of x-advection, the advection equation is given as

$$\frac{\partial(p\pm Zv_x)}{\partial t}\pm c\frac{\partial(p\pm Zv_x)}{\partial x}=0. \qquad (1)$$

In this equation, p is the sound pressure, v_x is the particle velocity, Z signifies the characteristic impedance (i.e. $Z=\sqrt{\rho K}$) and c represents the sound velocity in medium (i.e. $c=\sqrt{K/\rho}$), Here, ρ denotes the density of the medium, and K represents the bulk modulus.

In addition, through simple spatial differentiation of the equations, the equations of the derivatives are given as

$$\frac{\partial(\partial_x p\pm Z\partial_x v_x)}{\partial t}\pm c\frac{\partial(\partial_x p\pm Z\partial_x v_x)}{\partial x}=0. \qquad (2)$$

Figure 1 shows the sub-grid technique in the CIP method. Here, Δx and Δy represent the course grid size, while Δx_s and Δy_s are sub grid size, respectively.

Figure 2 shows the treatment of the boundary course grid and the sub-grid in propagation of $\pm x$ direction. In first step, we interpolate P, v_x, $\partial_x P$ and $\partial_x v_x$ in direction using Hermite interpolation. Next, we calculate advection equations (Eqs.(1) and (2)) in $\pm x$ direction. Notice that sub grid technique in the CIP analysis just needs to change interpolating function in sub grid region, because CIP scheme is based on a two-point stencil's MOC.

In this study, we use the GCIP scheme; GCIP(7,1), GCIP(3,1), and GCIP(3,0). Of these schemes, GCIP(7,1) and GCIP(3,0) respectively employ 7th-order Hermite interpolation and 3rd-order Lagrange interpolation with four stencils for the advection calculation.

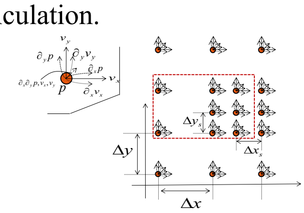


Fig. 1 Sub-grid technique in the CIP method.

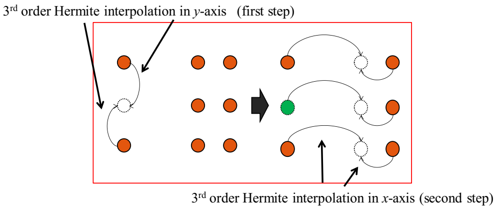
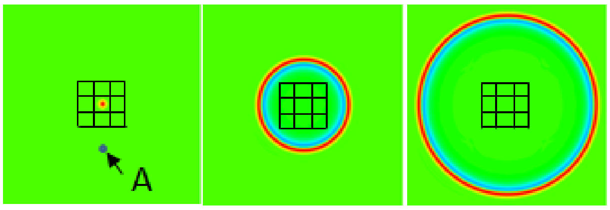


Fig. 2 Treatment of the boundary.



(a) $t = 10\,\Delta t$ (b) $t = 500\,\Delta t$ (c) $t = 1000\,\Delta t$

Fig. 3 Distribution of the sound pressure

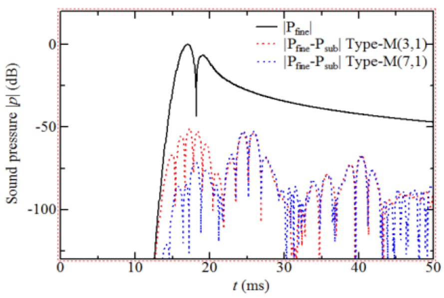


Fig. 4 Absolute pressure value: $|P_A^{fine}|$ and $|P_A^{fine} - P_A^{sub}|$

4. Results and discussion

We present numerical results obtained using the sub-grid technique in the CIP analysis. Calculation parameters are the following: the direction of acoustic field propagation, $\pm x, y$ (two-dimensional analysis); course grid size, $\Delta x = \Delta y = 0.06$ m; sub grid size, $\Delta x_s = \Delta y_s = 0.02$ m; time step, $\Delta t = 3.79\times10^{-5}$ s; ρ=1.21 kg/m^3 and $K = 1.42\times10^5$ Pa.

We also investigated the calculation time required for some sub-grid models. Here, we use a PC with Intel Core i7-980X Extreme Edition 3.33GHz. This processor has 6 cores and 12 hyperthreaded cores, or effectively scales 12 threads. For all analyses, parallel computation using OpenMP was applied.

Figure 3 shows the sound pressure distribution obtained using type-M CIP analysis with sub-grids at $t = 10\Delta t$, $t = 500\Delta t$ and $t = 1000\Delta t$, here GCIP(7,1) and GCIP(3,0) schemes are utilized for the advection calculation in course grids. The input pressure is driven from inside of the sub-grids. Here, the meshed area is the sub-grid region. We can ascertain the propagation behavior including that in the sub-grid region.

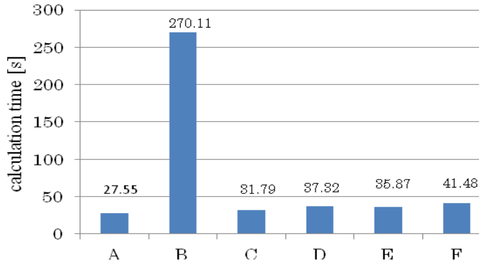


Fig. 6 Calculation time

Table . 1 Calculation parameter

	A	B	C	D	E	F
course grid;(0.06m)	800 * 800		800 * 800	800 * 800	800 * 800	800 * 800
fine grid;(0.02m)		2400 * 2400	100 * 100	150 * 150	100 * 100	150 * 150
CIP (3,1)	○	○	○	○		
CIP (7,1)					○	○

Figure 4 showed the error using sub-grids by means of comparison of the absolute pressure value at point A (see Fig. 3). We also show the numerical results obtained using the sub-grid technique for type-M CIP (i.e., GCIP31) analysis. Calculation parameters of both analyses are on equal terms. It is confirmed that the boundary in the sub-grids has good permeability characteristics with low reflection. The numerical error of the type-M GCIP(7,1) method is a little smaller than that of the type-M CIP method for acoustic simulation with a subgrid system.

Figure 5 shows the comparison of calculation time, where the calculation is divided into 500 time steps. Table 1 is the calculation parameter. The sub-grid model has a much shorter calculation time than the fine grid model. Fig. 5 also shows that CIP analysis with course grid that calculates with 7th-order Hermite interpolation required more calculation time than 3rd-order Hermite interpolation. This was because the number of variables for course grid is different.

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