Asymmetric-symmetric mode conversion of ultrasonic Lamb waves and negative refraction on thin steel plate

Jin Woo Sung^{\dagger} and Young H. Kim^{*} (Korea Science Academy of KAIST)

1. Introduction

Negative-index materials exhibit backward wave propagation, which means waves have group velocity direction antiparallel to phase velocity direction. They have been widely studied after the pioneering study by Lamb for both electromagnetic and acoustic waves [1].

Backward propagation of guided elastic waves (Lamb waves) is also well known and reported. Solution of Rayleigh-Lamb dispersion equation shows that in certain modes of Lamb waves, backward wave propagation occurs. However, the behavior of Lamb waves at interfaces which show backward propagation on one side and forward propagation on the other side only at a given frequency has not been reported until last year. Bramhavar et al. observed that Lamb wave originated from a point source focuses in another media [1]. They used a plate with two parts with different thicknesses such that forward-propagating symmetric mode is converted to backward-propagating symmetric mode at the selected frequency.

According to the dispersion equation of Lamb waves, anti-symmetric modes also exist. Exciting one side of a plate is likely to generate an antisymmetric mode [2]. If an anti-symmetric Lamb wave can be converted into a symmetric mode from a point source by negative refraction, it can be useful in generating symmetric Lamb wave modes.



Fig. 1 Dispersion curve of some modes of Lamb wave on a steel plate 0.78 mm thick. Lamb wave propagates backwards in S_{2b} mode.

corresponding author, E-mail: yhkim627@kaist.ac.kr

The goal of this research was to observe negative refraction of an anti-symmetric mode to a symmetric mode ultrasonic Lamb wave using a steel plate with a thickness change.

2. Experimental Details

The solution of dispersion equation gives several modes. **Fig. 1** is depicts dispersion curves of several Lamb wave modes. A dispersion curve's slope gives group velocity when wavenumber is the x-axis and angular frequency is the y-axis. The left part of S_1 mode has negative slope, meaning that backward wave propagation occurs. This part is called S_{2b} mode.

According to the dispersion equation, a Lamb wave mode occurs at a lower frequency for a thicker plate. For a single thickness, A_1 mode occurs below S_{2b} mode, but making h1 thinner than h2 allows two dispersion curves to intersect at a desired frequency. In this experiment, the steel plate was designed as shown in **Fig. 2(a)**. At 2.25 MHz, h1 part's A_1 mode and h2 part's S_{2b} mode intersects at wavenumber 0.93 mm⁻¹ as in **Fig. 2(b)**. A notable feature of the steel plate is that the thinner part is attached to the thicker part on one side of the plate. This design encourages an anti-symmetric mode to be converted into a symmetric mode easily.



Fig. 2 (a) Steel plate used in experiments, each part 5 centimeters long and wide. (b) Dispersion curve of parts overlapped, intersecting at 2.25 MHz frequency and 0.93 mm⁻¹ wavenumber.



Fig. 3 (a) Original waveform of Lamb wave received. (b) FFT results over entire waveforms. The distance of transmitter from interface is 1.5 cm (left), 3 cm (center), 4.5 cm (right). The distance of receiver from interface is 3 cm for all three experiments.

We used 2.25 MHz ultrasonic transducer with an acrylic block between the steel plate and the transducer with 10.8 degrees angle on one side to generated desired Lamb wave. The receiver (pin-type transducer) was fixed at 3, 4, and 5 cm from the interface on the thicker part and the transmitter was moved from 1.5 to 7.5 cm distance with 0.5 cm step from the interface on the thinner part. Both the transmitter and the receiver were located along the plate's line of symmetry.

We saved the waveform for each case and performed FFT (Fast Fourier Transform) on each waveform to find the peak voltage near 2.25 MHz to filter data by frequency. Then, voltage versus transmitter distance from interface was plotted to find evidence of negative refraction of Lamb wave by identifying focusing location.

3. Results and Discussion

Fig. 3(a) shows three sets of entire waveform data received by the pin-type transducer with sampling rate of 20MHz. From Fig. 3(a), it can be seen that Lamb wave disperses and arrives at the receiver over time. Lamb wave that propagates in the direction of the receiver arrives the earliest, but the amplitude is largest when Lamb wave that went through negative refraction arrives due to construct-ive interference. There is another time period with



Fig. 4 Peak values of FFT results classified by distance of receiver from interface.

large amplitude, which is reflection of Lamb wave at edges of the plate arriving at the receiver. Calculation of arrival time of negatively refracted Lamb wave at the receiver is about 60 microseconds after pulsation, which is equal to time where amplitude increases greatly in Fig. 3(a).

FFT results over entire waveform in **Fig. 3(b)** show peak at 2.25 MHz, which is the frequency of Lamb wave pulsated. The peak voltage indicates amplitude of Lamb wave at the receiver's location. There is also signal at about 1.8 MHz frequency, but it is low comparative to the desired signal. Amplitude after FFT increases and decreases as the transmitter is moved farther away from the interface. This indicates that there is a focal point along the center line.

Fig. 4 shows this trend more clearly. For the 3 cm data set, there is a peak at 2.5 cm transmitter distance. For the 4 cm and 5 cm data set, there is a peak at 3 cm transmitter distance. These values agree with locations of focal points calculated using Snell's law, although there are few data points to see the focal point moving away from the interface as the receiver is moved away.

In conclusion, we have observed data that suggests negative refraction occurs when an anti-symmetric Lamb wave is converted into a symmetric mode on a steel plate with change in thickness. This study can be applied to generate symmetric waves that appears to originate from a point source.

Acknowledgment

This work was supported by the Korea Foundation for the Advancement of Science and Creativity (KOFAC) grant funded by the Korean government.

References

- S. Bramhavar, C. Prada, A. A. Maznev, A. G. Every, T. B. Norris, and T. W. Murray: Phys. Rev. B. 83 (2011) 014106.
- 2. J. Lee, H. Kim, M. Choi, and Y. Kim: J. Acoust. Soc. Korea. **13** (1994) p. 50.