The relationship between Q value and temperature ratio for the step-shape thermoacoustic system

異なる内径を持つ熱音響システムにおけるQ値と温度比の関係討

Kenji Shibata^{1‡}, Shin-ichi Sakamoto², Yosuke Nakano³, Yoshiaki Watanabe³ (¹Grad. Sch. Eng., Doshisha Univ.; ²Dept. Elec. Sys. Eng., Univ. of Shiga Pref.; ³Grad. Sch. Sci. and Eng., Doshisha Univ.; ⁴Grad. Sch. Life and Med. Sci., Doshisha Univ.) 柴田 健次^{1‡}, 坂本 眞一², 中野 陽介³, 渡辺 好章⁴(¹同志社大・工;²滋賀県立大・工,³同志社大・理工, ⁴同 志社大・生命医科)

1. Introduction

Thermoacoustic systems^[1] are regarded as an external combustion engines that can be used exhaust heat from factories and electronic devices as a driving source. Consequently, thermoacoustic systems is one means to solve environmental problems. However, general thermoacoustic systems are too large to apply to electronic devices, in which miniaturized are required.

In our previous study, miniaturized systems with both ends closed were considered. Although the miniaturized system with uniform inner diameter (**Fig. 1(a**)) did not oscillate, the stable oscillation is successfully realized on the miniaturized system constructed by inner diameter tubes ^[2] (**Fig. 1(b**)).

We describe the functional mechanism the thermoacoustic system constructed by inner diameter tubes focus on the temperature ratio and the Q value as parameters to estimate the system loss.



- Fig. 1. Schematic of thermoacoustic system with both ends closed.
- (a) The system with uniform inner diamter.
 - (b) The system constructed by different inner diameter tubes.

2. Q value

The Q value^[1,3] is a parameter used to evaluate the loss in a system. The Q value can be expressed as in equation (1) using energy stored in a system E_s , the energy loss per unit time of a system \dot{E} , and resonance frequency ω .

ssakamot@mail.doshisha.ac.jp

$$Q = \omega \frac{E_s}{\dot{E}} \tag{1}$$

 \dot{E} is expressed as equation(2)^[1].

$$\dot{E} = \dot{E}_{pipe} + \dot{E}_{reg.} - \dot{E}_{\Delta T}$$
(2)

Therefore, equation (1) can be rewritten as equation (3).

$$Q = \omega \frac{E_s}{\dot{E}_{pipe} + \dot{E}_{reg.} - \dot{E}_{\Delta T}}$$
(3)

Therein, \dot{E}_{pipe} and \dot{E}_{reg} respectively denote the energy loss at the duct and regenerator per unit of time. $\dot{E}_{\Delta T}$ denote the energy production by the thermoacoustic effect per unit of time. The total acoustic energy E_s stored in the resonator is given as the volume integral of e_d ^[3].

$$E_s = \int e_d dV \tag{4}$$

 e_d is expressed as shown below^[3].

$$e_d = \frac{1}{4} \left(\frac{p^2}{\rho a^2} + \rho u^2 \right) \tag{5}$$

Therein, ρ and *a* represents the mean density and the adiabatic sound speed. *P* and *u* respectively represent the sound pressure and particle velocity. The acoustic energy density e_d is the sum of the acoustic potential-energy density and the acoustic kinetic-energy density. The energy loss $\dot{E}^{[3]}$ in the resonator is given by the total decrease of the acoustic power flow as

$$\dot{E} = \pi r_0^2 \bar{I}\Big|_{X=0}$$
 (6)

In that equation, r_0 is the inner diameter of the tube, and \bar{i} stands for the acoustic intensity of the tube, which represents the cross-sectional average of the dynamic power flux per unit area. X = 0 is origin at the left end of the system.

The Q value is a parameter that shows the vibration stability. So, more vibration is sustained when the Q value is large. That is, if the Q value

is small, then the loss in a system is large. Moreover, when a system is achieved the onset temperature ratio, the Q value diverges ^[1]. It is understood that the denominator is zero in equation (3). Therefore, this state is equivalent to 1/Q of zero.

3. Experimental setup

A schematic of the system is portrayed in Fig. 3. The total length of closed system was set to 2400 mm. A system with uniform inner diameter of 24 mm is compared to a system for which the combined tube inner diameter is 24 mm and the tube inner diameter is 42 mm. The length of a tube with inner diameter of 24 mm is defined as L_1 . The length of a tube with inner diameter of 42 mm is defined as L_2 . The system was set as L_1 : L_2 = 3:1, 2:1, and 1:1. The stack of honeycomb ceramics was produced with 0.65 mm channel radius, and thickness is 50 mm. To form a temperature difference in both stack ends, the electric heater is used. The input electric power was set to 30 W. The electric heater is set at 400 mm with the origin at the left end of the system to oscillate with half wave length. Pressure sensors (PCB Inc.) were used for sound pressure measurements. From the measured sound pressure in the tube, the distribution of sound intensity, sound pressure, and particle velocity are calculated using a Rott's equation^[4]. Q value is calculated from sound pressure distribution, particle velocity, and acoustic intensity flow by using equation(1).



Fig. 3. Experimental setup.

4. Results and Considerations

The plots of **Fig. 4** show experimental results of onset temperature ratio. T_H/T_C is 1.5 in the system with uniform inner diameter. On the other hand, T_H/T_C is 1.35 in the system of L_1 : $L_2=1:1$. This system is the lowest onset temperature ratio. We considered about this factor by using Q value. The relation between 1/Q and temperature ratio is presented in **Fig. 4**. When the temperature ratio is not formed, it turns out that 1/Q of a system that combines a different inner

diameter is large. This factor is expected that the loss in the connection area of the pipe is large. Moreover, when the temperature ratio is made to increase in each system, 1/Q is decreases because the energy conversion rises: the line slope expresses the grade of the energy conversion efficiency by increasing the temperature ratio. In spite of increasing loss, the slope of the line in the system with different inner diameter becomes steeper than that of the system with uniform inner diameter. Consequently, the factor of a low temperature dive in the system of L_1 : $L_2=1:1$ is because energy conversion efficiency is improved. So, the stable oscillation is successfully realized on closed tube type miniature thermoacoustic system in a low temperature ratio by setting L_1 : $L_2=1:1.$



Fig. 4. Relationship between 1/Q and temperature ratio.

5. Summary

This paper is described about a thermoacoustic system constructed by different inner diameter tubes, with emphasis on the loss and temperature ratio. Results show that low-temperature oscillation is enabled and that the energy conversion efficiency is improved by setting the tube lengths of different inner diameters to 1:1. In the future work, we will study on the optimum ratio of inner diameters.

Acknowledgements

This research was partially supported by a Grant-in-Aid for Young Scientists (A) (b), and a Grant-in Aid for Challenging Exploratory Research, the Program for Fostering Regional Innovation, and Specially Promoted Research of the University of Shiga Prefecture. **References**

- 1. A. Atchley: J. Acoust. Soc. Am. **95** (1994) 1661-1664.
- 2. Kenji Shibata *et al*.: J. Acoust. Soc. Am. Volume 131, Issue 4,(2012) pp. 3265-3265T.
- 3. T. Biwa, Y. Ueda, H. Nomura, U. Mizutani: Phys. Rew. E **72** (2005) 1-6.
- N. Rott and G Zouzoulas, Z. Angew. Math. Phys. 27 197–224 (1976).