Numerical simulation of a thermoacoustic engine – Effects of gas mixture on the onset temperature ratio –

熱音響エンジンの数値シミュレーション -混合気体が発振温度比に及ぼす影響-

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1. Introduction

Thermoacoustic systems, future energy systems, have been attracting attention because the drain problems of energy resources have surfaced. They have various styles: straight-tube type, loop-tube type, and various combinations of these types. At present, we study the straight-tube thermoacoustic engine. It is a system that can convert from heat to sound. Its energy conversion device comprises a low heat exchanger, a high heat exchanger and the stack, which is a porous device. When the temperature ratio of both ends increases through the use of two heat exchangers, the fluid in the system oscillates spontaneously. These systems are useful especially when driven by unused energy sources such as waste heat and solar heat because they are external combustion engines. Moreover, they are applicable as electrical generation and cooling systems. If low-temperature driving of the system can be achieved, then the systems can be put to practical use.

We investigated the onset temperature ratio of both ends of the stack to achieve low-temperature driving of the systems. The onset temperature ratio is the minimum temperature ratio for sound wave generation. To drive the system at low temperatures, it is necessary to assess the design because the onset temperature ratio is determined by the geometry and the working gas in the systems. It is reported that the onset temperature ratio decrease when gas mixture is used as working gas¹. However, it is not easy to change the gas and measure the onset temperature ratio, thus it has been studied only about a few gases in one system's geometry. If it can be calculated the onset temperature ratio of different gases and geometries by numerical simulation, it is more easy to find the system condition for low temperature drive.

In this report, the simulation methods of the onset temperature ratio in thermoacoustic engine filled with Ar and He gas mixture are described. As a result, it is confirmed that the calculated results agree with experimental results. By the numerical

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simulation, the onset temperature ratio in the system filled with gas mixture can be calculated.

2. Principle

2.1 Simulation methods

To estimate the acoustic field in the tube, transfer matrix method is $used^2$. The equation of motion and the equation for conservation of mass that were derived by Rott are used as governing equation. There are given as

$$\begin{bmatrix} P(L) \\ U(L) \end{bmatrix} = M_{(L,0)} \begin{bmatrix} P(0) \\ U(0) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} P(0) \\ U(0) \end{bmatrix} \quad (1)$$

where *P* and *U* respectively denote the sound pressure, the particle velocity. $M_{(L,0)}$ is the transfer matrix and depends on the temperature in the system, geometry, working gas and frequency. In this study, the particle velocities of the both ends (U(0), U(L)) are zero since both ends of thermoacoustic engine are closed. When m_{21} is zero, this condition is satisfied. Therefore, it determines the condition of stability limit in the straight tube, and the onset temperature ratio can be calculated.

2.2 Properties

In this calculation, density ρ , heat capacity ratio γ , constant pressure specific heat C_p , viscosity μ , and thermal conductivity λ are required as gas properties. The gas properties of pure Ar and pure He are well known, but the physical properties of gas mixture must be calculated. The calculation process of the physical properties is shown below³⁻⁴. Here, $R_i : R_j$ is mole fraction. N is molar weight. ϕ and ψ are combination coefficients of the viscosity and the thermal conductivity. S is the Sutherland constant number, which is calculable from the boiling point of individual gases. T is mean temperature

$$\rho_{ij} = \rho_i \cdot R_i + \rho_j \cdot R_j \tag{2}$$

$$\boldsymbol{\gamma}_{ij} = \boldsymbol{\gamma}_i \cdot \boldsymbol{R}_i + \boldsymbol{\gamma}_j \cdot \boldsymbol{R}_j \tag{3}$$

$$C_{pij} = \frac{C_{pi} \cdot R_i + C_{pj} \cdot R_j}{N_i \cdot R_i + N_j \cdot R_j}$$
(4)

$$\mu_{ij} = \mu_{i} \cdot \frac{1}{1 + \varphi_{ij} \cdot \left(\frac{R_{j}}{R_{i}}\right)} + \mu_{j} \cdot \frac{1}{1 + \varphi_{ji} \cdot \left(\frac{R_{i}}{R_{j}}\right)}$$
(5)
$$\left[\phi_{ij} = \frac{\left[1 + \left(\frac{\mu_{i}}{\mu_{j}}\right)^{\frac{1}{2}} \cdot \left(\frac{N_{j}}{N_{i}}\right)^{\frac{1}{4}}\right]^{2}}{2\sqrt{2} \cdot \left(1 + \frac{N_{i}}{N_{j}}\right)^{\frac{1}{2}}} \right]$$
$$\lambda_{ij} = \lambda_{i} \cdot \frac{1}{1 + \psi_{ij} \cdot \left(\frac{R_{j}}{R_{i}}\right)} + \lambda_{j} \cdot \frac{1}{1 + \psi_{ji} \cdot \left(\frac{R_{i}}{R_{j}}\right)}$$
(6)
$$\left[\psi_{ij} = \frac{1}{4} \left[1 + \left\{ \frac{\mu_{i}}{\mu_{j}} \cdot \left(\frac{N_{j}}{N_{i}}\right)^{\frac{3}{4}} \cdot \frac{1 + \left(\frac{S_{i}}{T}\right)}{1 + \left(\frac{S_{i}}{T}\right)} \right\}^{\frac{1}{2}} \right]^{2} \cdot \frac{1 + \left(\frac{S_{ij}}{T}\right)}{1 + \left(\frac{S_{i}}{T}\right)} \right]$$

3. Simulation model and experimental system

The experimental system is shown in Fig.1. Gas types are Argon gas and Ar:He (5:5) gas mixture. The cylindrical straight-tube has 2 m total length and 42.5 mm inner diameter. The stack has 50 mm total length and channel radius of three types (0.45, 0.55, and 0.65 mm). The stack position is 775 mm from the left end in the system. The simulation model is also the same condition.



4. Results and considerations

Experimental results and calculated results are shown in Fig.2. The calculated results agreed well with the experimentally obtained results. Therefore, it is confirmed that the onset temperature ratio in the system filled with gas mixture can be calculated by the numerical simulation. The calculated result of different mixture ratios is shown in Fig. 3. When Ar and He mixture ratio is 8:2 (channel radius of the stack is 0.6 mm), it is the minimum onset temperature ratio in this condition. This result is regarded as possible causes of Prandtl number and sound speed. When the Prandtl number decreases, the viscosity effect becomes smaller and heat exchange occurs more easily. Consequently, the energy dissipation decreases and the energy production increases. Then the thermoacoustic system can be driven with less energy production. As a result, the onset temperature ratio decreases. Furthermore, sound speed of argon gas is slower than helium. When the sound speed is low, there is less propagation loss of the sound wave. The amount of energy production is greater than the amount of the energy dissipation. Therefore, this thermoacoustic system can be driven with less energy production. As a result, the onset temperature ratio decreases. We conclude that the onset temperature ratio decreases because of these two factors.



Fig. 2. The Onset temperature ratio of Ar gas and Ar:He gas with the change of the stack channel radius.



Fig. 3. The onset temperature ratio of the calculated results with different mixture ratios.

5. Summary

The simulation methods of the onset temperature ratio in thermoacoustic engine filled with gas mixture were described. As a result, it is confirmed that the calculated results agree with experimental results. By the numerical simulation, the onset temperature ratio in the system filled with gas mixture can be calculated. In the future, it is possible to research the onset temperature ratio in the system filled with different gas mixtures.

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