

Propagation of Sound Wave in Rarefied Polyatomic Gases Based on Extended Thermodynamics

II. Transverse Wave

拡張された熱力学に基づく多原子分子希薄気体中を伝播する音波の解析 II. 横波

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1. Introduction

Extended thermodynamics (ET) [1] is a theory which is applicable to highly-nonequilibrium phenomena beyond the validity range of the theories based on the assumption of local equilibrium, such as Navier-Stokes and Fourier (NSF) theory. Recently ET theory of rarefied polyatomic gases and also dense gases was developed [2]. The validity and the usefulness of that theory are confirmed by the fact that the ET theory can explain the dispersion relation of longitudinal waves in rarefied polyatomic gases even in the high-frequency region in which the NSF theory fails [3].

The purpose of the present paper is to make clear the experimental condition to measure transverse waves in rarefied polyatomic gases quantitatively. In the NSF theory, there is no solution of propagating transverse waves because their governing system of partial differential equations is parabolic type. However, there can exist the solution in the ET theory which has a system of hyperbolic balance equations. As the ET theory predicts the same result as that of the NSF theory in low-frequency region, we will focus on the dispersion relation in the high-frequency region.

2. Basic equations

We adopt the ET theory with 14 independent variables; the mass density ρ , velocity v_i , temperature T , symmetric traceless part of viscous stress $S_{\langle ij \rangle}$, dynamic pressure $\Pi (=S_{ii}/3)$ and the heat flux $q_i (i,j=1,2,3)$. The caloric and thermal equations of state for rarefied polyatomic gases are given by

$$\varepsilon = \varepsilon(T), \quad p = a\rho T,$$

where ε and p are, respectively, the specific internal energy and the pressure. Here a is given by k_B/m with k_B and m being the Boltzmann constant and the mass of a molecule. The specific form of ε is determined, for example, by quantum statistical mechanics.

The basic equations are the linearized equations in the neighborhood of a reference equilibrium state:

$$\dot{\rho} + \rho_0 \frac{\partial v_k}{\partial x_k} = 0,$$

$$\rho_0 \dot{v}_i + aT_0 \frac{\partial \rho}{\partial x_i} + a\rho_0 \frac{\partial T}{\partial x_i} - \frac{\partial S_{\langle ij \rangle}}{\partial x_j} + \frac{\partial \Pi}{\partial x_i} = 0,$$

$$a\rho_0 c_v^* \dot{T} + a\rho_0 T_0 \frac{\partial v_k}{\partial x_k} + \frac{\partial q_k}{\partial x_k} = 0,$$

$$\dot{S}_{\langle ij \rangle} - 2a\rho_0 T_0 \frac{\partial v_{\langle i}}{\partial x_{j \rangle}} - \frac{2}{1+c_v^*} \frac{\partial q_{\langle i}}{\partial x_{j \rangle}} = -\frac{1}{\tau_S} S_{\langle ij \rangle},$$

$$\dot{\Pi} + \left(\frac{2}{3} - \frac{1}{c_v^*} \right) a\rho_0 T_0 \frac{\partial v_k}{\partial x_k} + \frac{2c_v^* - 3}{3c_v^*(1+c_v^*)} \frac{\partial q_k}{\partial x_k} = -\frac{1}{\tau_{\Pi}} \Pi,$$

$$\dot{q}_i + (1+c_v^*) a^2 \rho_0 T_0 \frac{\partial T}{\partial x_i} - aT_0 \frac{\partial S_{\langle ik \rangle}}{\partial x_k} + aT_0 \frac{\partial \Pi}{\partial x_i} = -\frac{1}{\tau_q} q_i,$$

where a quantity with the subscript 0 represents the quantity in the reference equilibrium state and the summation convention is adopted. Here c_v^* and x_i are, respectively, the dimensionless heat capacity $(d\varepsilon/dT)_0/a$ and the position. A dot on a quantity represents material time derivative, and τ_S , τ_{Π} , τ_q are the relaxation times which are related to the shear viscosity μ , the bulk viscosity ν and the heat conductivity κ , respectively.

3. Dispersion relation for transverse waves

We consider a plane harmonic wave propagating in x_1 -direction. Without loss of generality, we can assume the following type of the solution of a transverse wave:

$$v_i \equiv \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \quad S_{\langle ij \rangle} \equiv \begin{pmatrix} 0 & S & 0 \\ S & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad q_i \equiv \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix}.$$

Note that we can easily prove that $\rho = \rho_0$, $T = T_0$ and Π must be zero. The solution is expressed, with frequency ω and complex wave number k , by

$$\mathbf{u} = \mathbf{w} e^{i(\omega t - kx_1)},$$

where u , w and t are, respectively, the state vector $(v_i, S_{\langle ij \rangle}, q_i)$, a constant amplitude vector and time.

The dispersion relation is obtained by

$$-aT_0 \left(\frac{2+c_v^*}{1+c_v^*} - \frac{i}{\Omega\tau_{qs}} \right) z^2 + 1 - \frac{1}{\Omega^2\tau_{qs}} - i \frac{1+\tau_{qs}}{\Omega\tau_{qs}} = 0,$$

where $\Omega = \tau_s \omega$, $\tau_{qs} = \tau_q / \tau_s$, $z = k / \omega$. The phase velocity v_{ph} and the attenuation factor α are expressed in terms of the solution z of the dispersion relation:

$$v_{ph}(\omega) = \frac{\omega}{\text{Re}(k)} = \frac{1}{\text{Re}(z)},$$

$$\alpha(\omega) = -\text{Im}(k) = -\omega \text{Im}(z).$$

4. Typical results: the case of n-H₂

We study the dispersion relation numerically. For example, the results in the case of normal hydrogen are shown. The results for other kinds of gases will be reported at the conference. We can estimate the temperature dependence of τ_{qs} by using the experimental data on μ and κ [3]. The conditions of the numerical analysis are as follows: $T_0 = 293$ [K], $c_v^* = 2.45$ and $\tau_{qs} = 1.46$.

Figure 1 shows the dependence of the dimensionless phase velocity v_{ph}/c_0 and the attenuation factor $c_0\tau_s\alpha$ on the dimensionless frequency Ω , where c_0 is the (longitudinal) sound velocity in the low-frequency limit in the reference equilibrium. It can be seen that both v_{ph}/c_0 and $c_0\tau_s\alpha$ approach zero as Ω tends to zero. When Ω becomes large, these have nonzero values and approach some finite values as Ω tends to infinity.

Figure 2 shows the ratio between the characteristic length $\delta = 1/\alpha$, which is the propagation length that the amplitude becomes $1/e$ times, and the wavelength λ . This ratio represents how quickly the amplitude decreases compared with the wavelength. In order to observe a vibrating character of a wave, the value of this ratio must be larger than $O(10^0)$. It is seen from Figure 2 that the

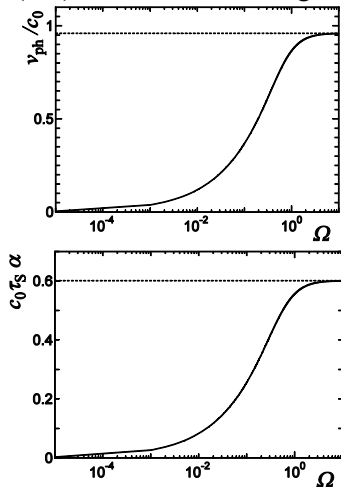


Fig. 1 Dependence of the dimensionless phase velocity (above) and attenuation factor (below) on the dimensionless frequency Ω for n-H₂ at 293[K]. Horizontal dotted lines indicate the limiting values when $\Omega \rightarrow \infty$.

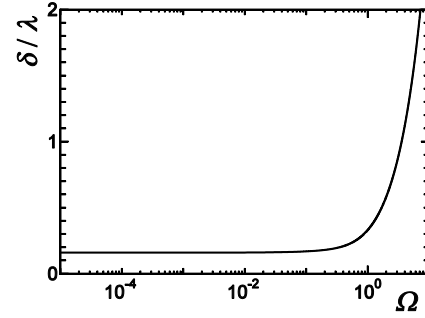


Fig. 2 Dependence of ratio between the characteristic length $\delta (= 1/\alpha)$ and the wavelength λ on the dimensionless frequency Ω for n-H₂ at 293 [K].

ratio has very small value in the low-frequency region and increases monotonically with the increase of Ω . The above necessary condition becomes to be satisfied when Ω is higher than $O(10^0)$. We conclude that Ω should be higher than $O(10^0)$ for experimental observations of the transverse waves.

5. Summary and concluding remarks

In this paper, the experimental condition to measure transverse waves in rarefied polyatomic gases quantitatively is obtained based on the recently-developed ET theory.

The conclusion is as follows: It is difficult to detect the transverse waves in the low-frequency region in which the NSF theory is valid. However, the transverse waves are measurable in the high-frequency region with Ω higher than $O(10^0)$. We expect the experiments for high-frequency transverse waves, for example, transverse waves of $\Omega = 5.0$ which corresponds to $\omega \sim 0.5$ [GHz] at the pressure $p_0 = 1000$ [Pa].

Because the wavelength of a transverse wave is very small and because the wave decays quickly, the results may play important roles in the phenomena in small scale, for example, those in nano-technology.

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