# **Rectification of elastic waves in nanowires**

ナノワイヤーにおける弾性波の整流

Daichi Kono<sup>‡†</sup>, Yukihiro Tanaka<sup>\*</sup>, and Norihiko Nishiguchi Division of Applied Physics, Graduate School of Engineering, Hokkaido University 河野大地<sup>‡†</sup>, 田中之博<sup>\*</sup>, 西口規彦 (北大院工)

## 1. Introduction

A diode is an essential passive device that rectifies an electric current, and the device is indispensable to any circuits. On the other hand, a device that rectifies acoustic waves or vibrations like the diodes has not been realized. In Refs.[1-2], we suggested a rectification mechanism for bulk acoustic waves. The system (as shown in Fig.1 with  $d=\infty$ ) makes it possible to rectify the bulk acoustic waves, utilizing both geometric effects on acoustic wave scattering due to the asymmetric scatterers and interference effects among the scattered waves. We numerically confirmed the rectification of bulk acoustic waves with a finite-difference time-domain (FDTD) method. However, it is not easy to actually fabricate the device since it must be very thick so that surface effects on the rectification of bulk waves are negligible.

We also investigated acoustic wave transmission through the scatterers in a *thin layer* containing the same periodic arrangement of triangular scatterers as shown in Fig.1 with a finite d in thickness, and confirmed numerically the rectification effects, too, in which the acoustic waves are symmetric and anti-symmetric Lamb waves [3]. But the system extends in the *x*-*y* plane, and then the system is not compact either.

In this work, we present a feasible compact acoustic rectifier, which is a beam containing a single triangular void as shown in Fig. 2, and confirm numerically the rectification of acoustic waves.



Fig.1. Model of acoustic wave rectifier in Refs. [1-3]. The rectifier is made of an elastically isotropic material of thickness  $d=\infty$  for Ref. [1-2] and  $d<\infty$  for Ref. [3] and the triangular voids are equally spaced in the *y* direction.

## 2. Model and methodology

The present model (Fig.2) is composed of an isotropic material (tungsten) with a single triangular hole. The structural parameters including the cross sectional dimensions are given in Fig.2, which are measured in unit of the base length *a* of the triangle.

The model is obviously the same as a unit cell of the system shown in Fig.1 with a finite d. Considering acoustic waves are reflected at the free side sections, we may expect the same rectification performance as Ref. [3]. However, the phonon modes in the beam structure, which are derived precisely in Ref. [4], are different from those in the bulk material or in the thin layer. In this work, we investigate the acoustic wave rectification in the system.

The equation of motion governing the displacement vector field  $u_i(\mathbf{r},t)$  and the stress tensor  $\sigma_{ii}(\mathbf{r},t)$  are given by

$$\rho(\mathbf{r})\ddot{u}_{i}(\mathbf{r},t) = \partial_{j}\sigma_{ij}(\mathbf{r},t) \quad (i = 1,2,3),$$
  
$$\sigma_{ij}(\mathbf{r},t) = c_{ijkl}(\mathbf{r})\partial_{l}u_{k}(\mathbf{r},t) \quad (i,j = 1,2,3),$$

where  $\mathbf{r} = (x, y, z)$  and  $\rho(\mathbf{r})$  and  $c_{ijkl}(\mathbf{r})$  are the position-dependent mass density and stiffness tensor of substrate, respectively.

We numerically integrate the equations and obtain  $u_i(\mathbf{r},t)$  and  $\sigma_{ij}(\mathbf{r},t)$ , from which we derive the Fourier components of displacement vector  $\hat{u}_i(\mathbf{r},\omega)$  and stress tensor  $\hat{\sigma}_{ij}(\mathbf{r},\omega)$ . Using them, we evaluate the acoustic Poynting vector, whose *x*-component is given by

$$\hat{J}_{x}(x,\omega) = -4\pi \iint \operatorname{Im}\left[\omega \hat{u}_{j}(\mathbf{r},\omega)\hat{\sigma}_{jx}^{*}(\mathbf{r},\omega)\right] dy dz.$$

Here we define the transmission rate  $T(\omega)$  by the ratio of  $\hat{J}_x(x_D, \omega)$  to that in the absence of hole  $\hat{J}_x^0(x_D, \omega)$ ;

$$T(\omega) = \frac{\hat{J}_x(x_D, \omega)}{\hat{J}_x^0(x_D, \omega)},$$

where  $x_D$  represents the detecting position behind the scatterer. In the following, we refer to as case

<sup>\*</sup>Corresponding author: yuki@eng.hokudai.ac.jp



Fig.2. Model of rectifier in this work. The model also has triangular holes and is subject to the free boundary condition (FBC) and the absorbing boundary condition (ABC). The FBCs are imposed on both the surfaces normal to the y and z directions along the array, and the ABCs are used on both the end surfaces normal to the x direction. S and D indicate the cross section for excitation and detection of acoustic waves when the incident waves impinge first on the summits of the scatterer case (1). The positions are switched for case (2).

(1) when the incident waves propagate in the x-direction, and to as case (2) for propagation in the opposite direction.

#### 3. Numerical Results

In our simulations, we use a wave packet made of z-polarized flexural vibrations for evaluation of transmission rates. The wave packet has a Gaussian frequency distribution having the central frequency at  $\omega_0 a/v_t = 5\pi/2$  with the width  $\Delta \omega a/v_t = 2\pi$ . We examine the transmission rates when the wave packet is excited at  $\delta = 10a$ , where  $\delta$  is the distance from the scatterer. Figure 3(a) shows the transmission rates versus frequency. The thick and thin lines indicate the transmission rates for cases (1) and (2), respectively. The transmission rates depend on the propagation directions in the frequency region  $\omega a/v_t > \pi$ , showing the obvious difference in magnitude. Thus, we confirm that the system rectifies acoustic waves. Figure 3(b) also plots the transmission rate versus frequency, showing that the disagreement of transmission rates mostly occurs in the frequency region  $\omega a/v_t > 10$  in contrast to Fig.3(a). The initial condition for Fig.3(b) is different from Fig.3(a), where the wave packet is excited at  $\delta = 5a$ .

Despite that the rectification of acoustic waves are confirmed in both Figs.3(a) and (b), the frequency dependences are not consistent, in contrast to the previous cases [1-3]. The position dependence of transmission rates stems from that the waves repeatedly bounce at the two free side sections of the beam. Because of the property, it is



Fig.3. Transmission rates versus frequency when *z*-polarized flexural waves enter toward a triangle void. The incident wave packet has central frequency  $\omega_0 a/v_t = 5\pi/2$  and the width  $\Delta \omega a/v_t = 2\pi$  in frequency domain and the distances  $\delta$  between the triangular void and the position where the incident wave packet is initially excited are (a)  $\delta = 10a$  and (b)  $\delta = 5a$ . Thick and thin solid lines represent case (1) and case (2), respectively.

possible to change the transmission rate by adjusting the excitation position of waves, which will lead to various uses of the acoustic rectifier.

### 4. Concluding Remarks

To conclude, the rectification of acoustic waves takes place in the beam structure with a single triangular hole, and the rectification effects are sensitive to the position of excitation of the incident wave packet. Utilizing the properties, it is possible to make a compact acoustic-wave rectifier.

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